Empirical Investigation of Multiclass Vehicle Behaviour under Heterogeneous Traffic Flow Conditions

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Abstract

Empirical data is the basis for development and validation of traffic flow models. From the videographic data collected on two urban arterial sections, the temporal evolution of macroscopic variables such as speed, flow and density have been established. Speed data has been obtained to determine the class specific behaviour with respect to varying flow conditions and lane positions. From the statistical analysis, it is inferred that modelling using mean speed values of the vehicles alone is not sufficient, standard deviation of speeds for changing density also needs to be taken in to account.High degree of variation in speeds are observed for fast moving vehicles(Cars and Motorised Two Wheelers) compared to slow moving vehicles (Heavy vehicles and Motorised Three Wheeler) which shows that heterogeneous traffic on arterial consists multiple classes of drivers, where the distribution of these driver classes are more in stationary condition than in non-stationary condition.The analysis also revealed that lane positioning has a significant effect on vehicular speeds. However, thereis a little variance of speeds across the lanessuggestvehicle dynamics can be described by considering the whole width of the road as a single unit instead of separate entities. The results of the study show that,if multiple classes of vehicle-driver units and their interactions are considered, then the macroscopic models would be able torepresent traffic dynamics in a better way.

Keywords:Heterogeneous traffic, Vehicular Speed, Multiclass vehicles, Multiclass drivers, ANOVA

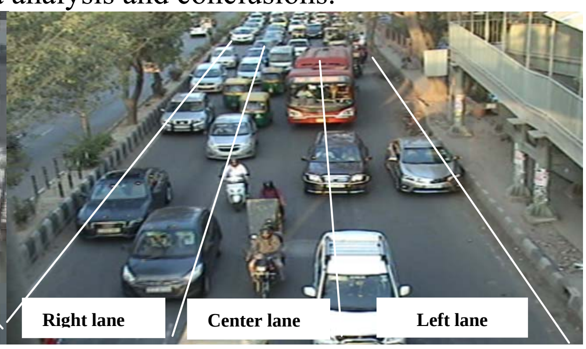
1. Background

Several macroscopic models have been developed to capture complex vehicular dynamics on various traffic flow facilities with the aim of improving existing traffic flow conditions. Initial developments such as first order models are mainly focused on studying shock wave propagation and queue length estimation (Lighthill & Whitham, 1955,Michalopoulos et al., 1984). Subsequently, higher order differential equations replace the first order models to overcome various draw backs such as infinite speed and acceleration with in the shock. Moreover, they help in reproducing some of the traffic phenomena such as stop and go waves, platoon dispersion, hysteresis phenomenon and others (Payne, 1979, Bando et al., 1995,Treiber et al., 1999, Aw & Rascle, 2000, Zhang, 2002). The higher order models further evolved by considering multiclass drivers and vehicles in their mathematical formulations (Hoogendoorn & Bovy, 2000, Helbing et al., 2001, Tang et al., 2009). However, researchers proved that it is possible to replicate some of the complex traffic phenomena by introducing multiclass drivers, multiclass vehicles and their complex interactions into first order models

(Wong & Wong, 2002, Logghe & Immers, 2003,Chanut& Buisson,2003, Ngoduy, 2011). Besides, some models describe the road dynamics in an overall manner and others describe each lane as a separate entity. But these developments are limited to lane based traffic environments. In this regard there is a growing interest in modelling non-lane based heterogeneous traffic systems due to their complexity and uniqueness. For instance, Nair et al.(2011) proposed porous flow approach by considering probability distribution of gaps accepted by vehicles in the traffic stream and it closely resembles the behaviour of heterogeneous traffic. Except for this model, there is no considerable effort in modelling heterogeneity, especially in Indian traffic condition. Questions are still prevailing in understanding the behaviour of non-lane based heterogeneous streams where the behaviour of vehicles concerning various flow conditions and lane positioning is hardly known.

Stream speed plays an important role in assessing the level of service offered by a roadway facility and understanding the vehicle-driver behaviour under different circumstances. It is also a major component in mathematical models used in estimating travel times and congestion propagations. The speed of a particular vehicle class depends on its static and dynamic characteristics, psycho-physical characteristics of the drivers and the surrounding environment. Speed selection by the drivers in non-lane heterogeneous traffic environment such as presented in India (Figure 1(b) and 1(c)) is the contrast to the homogenous traffic environment. The behaviour of heterogeneous traffic streams is complex where stream movement is greatly influenced by the presence of Motorised Two Wheelers (MTW) and Motorised Three Wheelers (MThW). Their unique behaviour such as high manoeuvrability and small cross-sectional area (area occupancy) increases the uncertainty in traffic movement thereby make model development critical. Few studies have been carried out in Indian traffic conditions to evaluate vehicle dynamic characteristics such as speed distributions (Dey et al. 2006), lateral and longitudinal placement (Chunchu et al. 2010), headway distributions (Sharma et al. 2011, Dubey et al. 2012) and acceleration/deceleration, selection of lateral spacing and longitudinal distances (Kanagaraj et al. 2015). However, these studies are limited to evaluating microscopic flow characteristics for given traffic conditions. The present study focuses on evaluating the aggregate behaviour of different classes of vehicle-driver units under various traffic flow conditions on busy urban arterials. Further, it concentrates on studying the vehicular behaviour with respect to the choice of lane selection. A systematic and detailed statistical procedure has been adopted to solve the above research problem. The study gives some interesting results. It is found that MTW have significant advantage in terms of mobility over other classes of vehicles across the different spectrum of densities. Large variations in speeds were observed for fast moving vehicles such as MTW and Cars compared to MThW and Heavy Vehicles (HV). Lane positioning also has a significant impact on vehicle speeds.The remaining sections of this paper cover methodology adopted for this study; data collection procedure, data analysis and conclusions.

(a)



(b)

Figure 1: Traffic (a) at near capacity condition - Panchsheel Marg (b) at congested condition -Ho-Chi-Minh Marg

2. Methodology

This study aims to answer few questions such as the effect of flow condition on vehicular and driver behaviour, distribution of driver classes; effect of lane position on vehicle speeds. To answer these questions following methodology has been adopted.

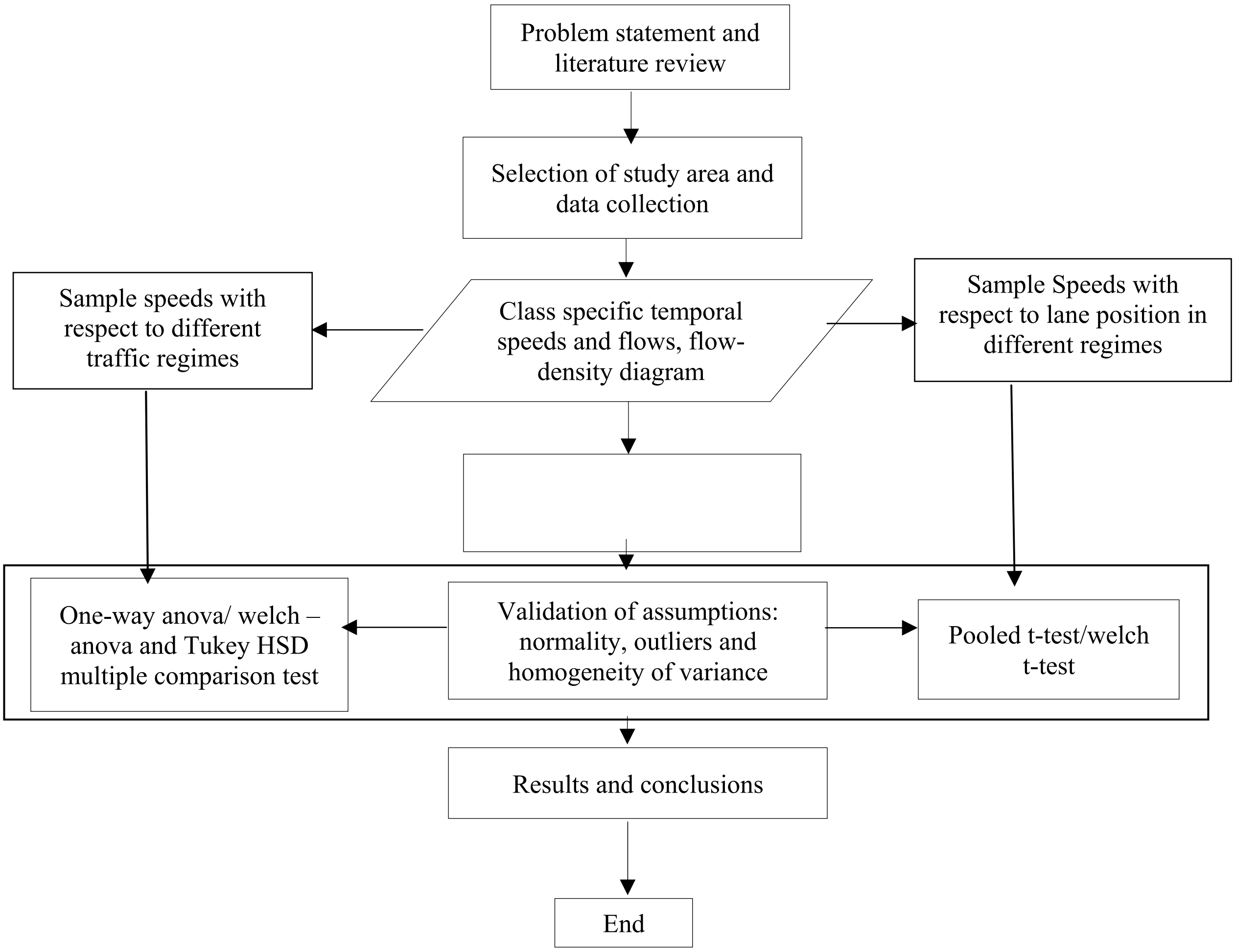


Figure 2: Methodology for data collection and statistical analysis

3. Data collection and analysis

Data has been collected on two study sections Panchsheel Marg (28°32'35.5"N, 77°12'45.8"E) and Ho-Chi-Minh Marg (28°32'37.3"N, 77°14'39.9"E) located on outer ring road urban arterial, New Delhi, India (Figure 1 (a) and Figure 1(b)). In these study locations, roadway widths are uniform (i.e. 10.5 m) with flat gradients. Based on physical and dynamic characteristics, vehicles are classified in to four types namely Cars, MTW, MThW and HV. Approximately 10 hours of data (9:30 a.m. to 5:30 a.m. at Panchsheel Marg and 4:30 p.m. to 6:00 p.m. at Ho-Chi-Minh Marg) was collected on 3rd march 2016 and 9th march 2016 using video cameras. Class specific flow and speed values were extracted using Trazer® software (Mallikarjuna et al. 2009). Consequently, number of vehicles passed the road section Q_i (x,t) and average speed $\overline{V_i}(x,t)$values of a specific vehicle class were calculated for a time interval (ΔT) of one minute using Equations (1) to (5) and the time evolution of these plots are shown in Figure 3, 4 and 5. The maximum flow rate and maximum speed observed on these sections are 11,760 veh/h and 70 km/h respectively.

Class specific flow: $$Q_i(x,t) = \frac{N_i(x,t)}{\Delta T} \quad (1)$$

Total Flow: $$Q(x,t) = \sum_{i=1}^{n} Q_i(x,t) \tag{2}$$

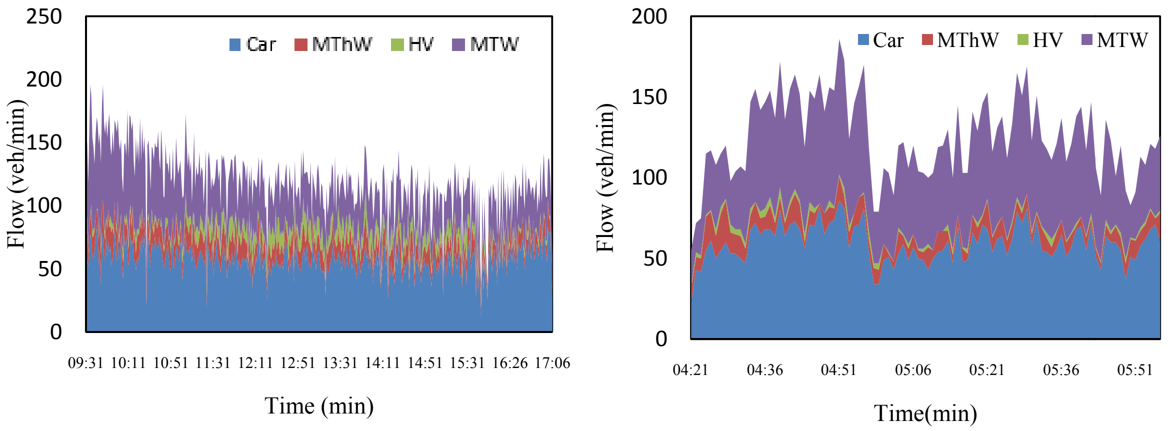
Class specific average speed: $$\bar{V}_i(x,t) = \sum_{j=1}^{m} \frac{m}{\frac{1}{V_j}} \tag{3}$$

Average stream speed: $$\bar{V}(x,t) = \sum_{i=1}^{n} P_i \, \bar{V}_i(x,t) \tag{4}$$

Total density is determined using the fundamental relation:

$$Q(x,t) = K(x,t)\bar{V}(\mathrm{x},\mathrm{t}) \tag{5}$$

Where i, j, P_i represents the number of vehicle classes, number of vehicles present in each vehicle group and percentage of vehicles in total number respectively. The density of the stream is determined using Equation 5 and it is represented in the number of vehicles/km/meter width of the road.



(a) (b)

Figure 3: Temporal flow variation (a) at Panchsheel Marg (b) at Ho-Chi-Minh Marg

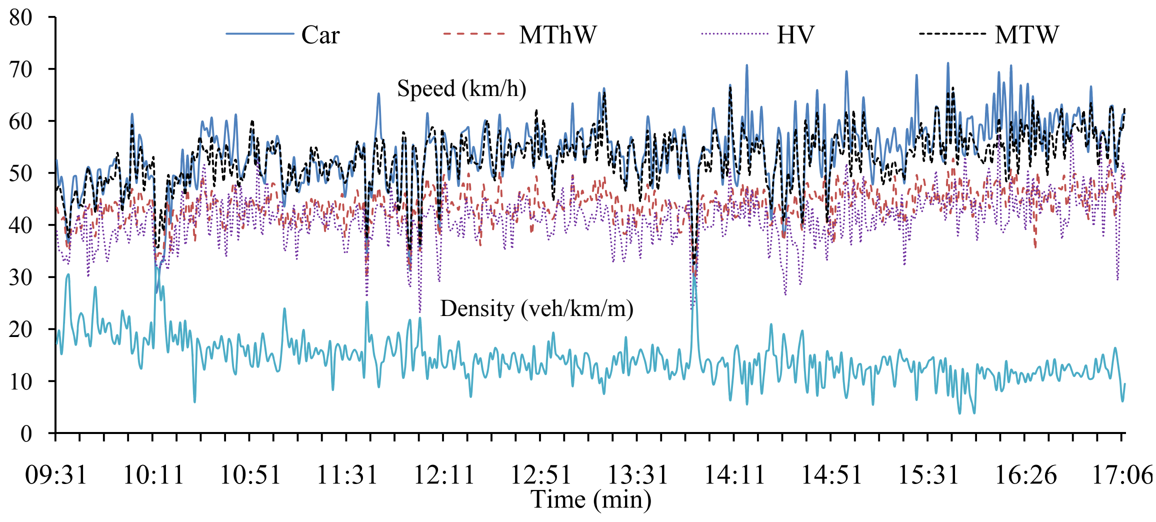


Figure 4: Temporal Speed variation of vehicle classes at Panchsheel Marg

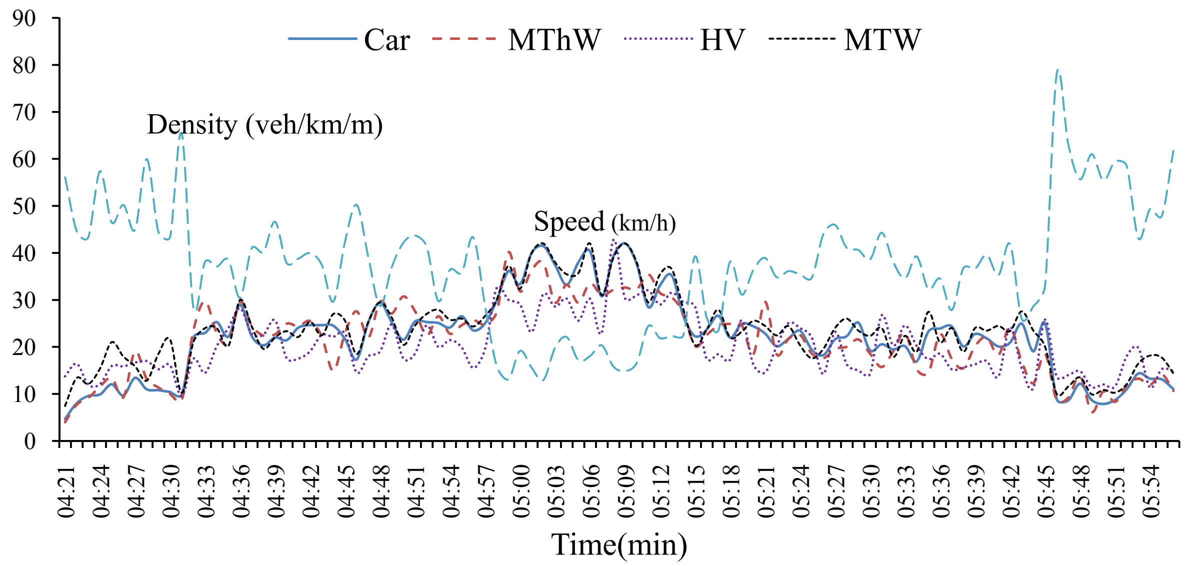


Figure 5: Temporal Speed variation of vehicle classes at Ho-Chi-Minh Marg

In total traffic volume it is observed that, cars and MTW's occupy greater percentage than vehicles such as MThW and HV. It is clear from the speed profiles (Figure 4) that cars and MTW's are maintaining higher speeds than vehicles such as MThW and HV but behave identically (acceleration and deceleration) in the stationary conditions. Considering this, vehicles can be grouped in to fast moving (Cars and MTW) and slow moving (MThW and HV) vehicles. In congested conditions (Figure 5) less variation in speeds was observed between the vehicles. It is believed that presence of different classes of drivers in traffic stream leads to the generation of stationary and non-stationary conditions in the flow-density fundamental diagram (Figure 6). Further investigationsare needed to confirm the distribution of driver classes accurately in various density regions.

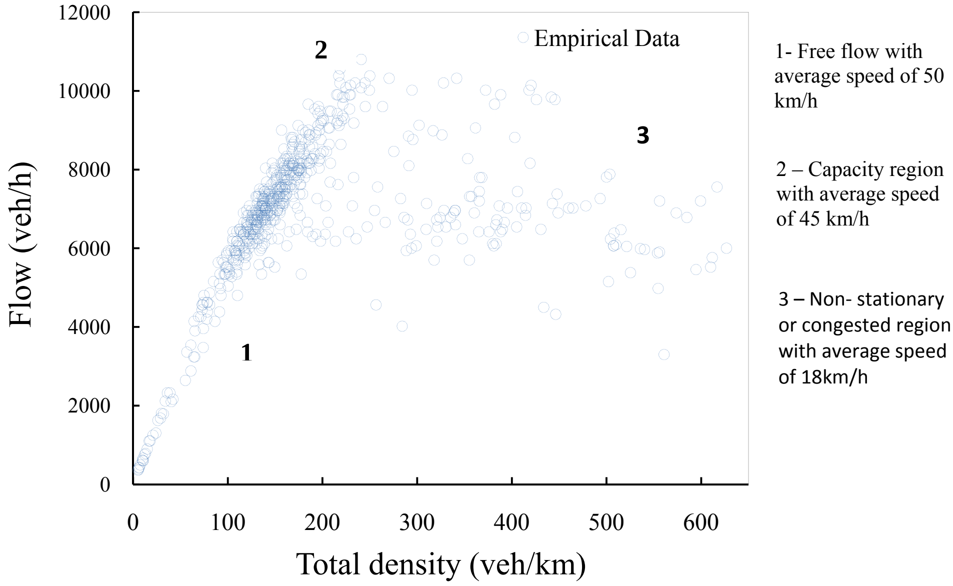


Figure 6: Flow-Density relationship for heterogeneous traffic stream

In the subsequent sections, statistical tools were used to confirm the variation in the behaviour of vehicle classes across different flow regimes. Moreover, it also reveals some interesting facts about how drivers of specific vehicle class are selecting their speeds with respect to the flow conditions. Different flow regimes are identified namely free flow, capacity and congestion from various plots such as temporal speed, flow and flow-density (Figure 6). Moreover, to study the impact of vehicle position on the speeds, entire section of road width is divided into 3 regions namely rightlane, center lane and leftlane as shown in Figure 1.A detailed analysis procedure and the results will be discussed under sections 3.1 and 3.2. Statistical analysis was carried out using R statistical software (R Core Team, 2015).

3.1 Impact of flow condition on speed

A null hypothesis has been framed to study the effect of flow condition on the behaviour of vehicle and driver classes. The null hypothesis is that there is no significant difference in average speeds observed between the vehicle classes at a given traffic condition and any difference observed is due to a random effect. It is achieved by applying the one-way analysis of variance (one way ANOVA) statistical test to check the differences between the groups. The selection of the test and the credibility of the outcomes depend on the significance of the test assumptions. The first assumption is the test of normality which was satisfied by the large sample size in the present study. Also, the assumption is validated by quantile-quantile plots[1] (Q-Q plots), skewness[2] and kurtosis[3].Further,speeddata in all the flow regions does not satisfy with the homogeneity of variance[4] (Bartlett's test) assumption and the outliers assumption[5] (Bonferroni test). Therefore, modified ANOVA test such as Welch-Anova has been adopted for the analysis (where correction to the F-test value will be done) and Tukey HSD test was chosen for multiple comparisons.

From the Welch-ANOVA test results and descriptive statistics (Table 1) it is found that, there is a significant difference in average speeds adopted by different vehicle groups at free flow and capacity regions. Mean plots with 95% confidence intervals (Figure 7(a) and 8(a)) depict the data dispersion amongst the different vehicle classes. Vehicles such as cars and MTW's are travelling at higher speeds than the MThW's and HV's. High degree of variation in speeds represents freedom in selecting the speeds and the presence of different driver classes.

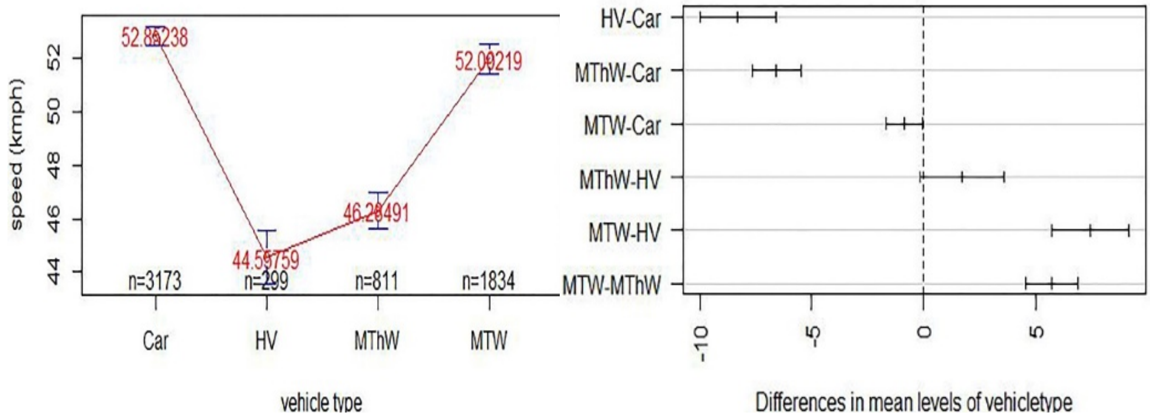
It concludes modelling using mean speed values of the vehicles alone is not sufficient, the standard deviation of speeds for changing density also needs to be taken in to account. For example, while developing speed-density equilibrium relationship, the variance must be included. Probably, driver classes can also be delineated using standard deviation values. Tukey-HSD multiple comparison results (Table 1, Figure 7(b) and 8(b)) reveals that the vehicles can be grouped in to fast moving and slow moving in terms of selecting their speeds. Modelling can be done with only two vehicle classes to avoid complex dynamics and to reduce computation time however the model accuracy will be compromised.

Further, the data analysis in the congested region revealed that the vehicle classes behave almost identical as the variance around the mean decreases showing the presence of fewer driver classes. It is due to restricting the freedom of movement laterally and longitudinally. Expectedly, MThW's are moving at a slower pace than other vehicles (Figure 9).

Table 1: Anova statistical analysis between vehicles observed in different flow conditions

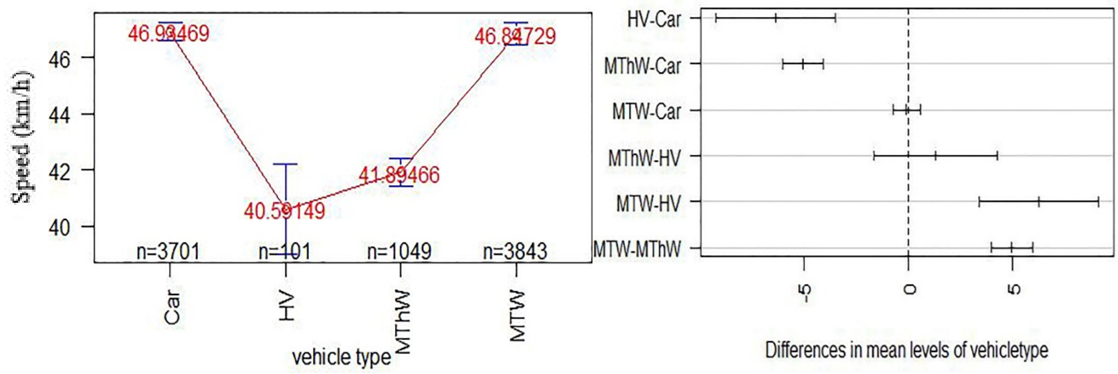
Time/ type of flow	Vehicle type	Sample size	Mean speeds (km/h)	Median speeds (km/h)	σ* (km/h)	S.E. * mean	Skewness	Kurtosis	Bartlett's test p-value	Bonferonni Outliers test p-value	Welch -Anova p-value	Tukey HSD multiple comparison
1:00 p.m. to 2:00 p.m. (free flow)	Car	3172	52.85	52.99	10.51	0.19	-0.22	1.34	<2.2 e-16	0.025	<2.2 e-16	Except MTW-Car (p-value = 0.04), MThW-HV (p-value = 0.09) all are significantly different in mean speeds
	HV	298	44.59	45.19	8.83	0.51	-0.83	1.64				
	MThW	810	46.28	47.81	9.85	0.35	-1.19	1.99				
	MTW	1834	52.02	52.5	12.03	0.30	1.71	1.11				
9:30 a.m.to 10:30 a.m. (capacity)	Car	3700	46.94	47.19	10.25	0.17	-0.43	1.81	<1.8 e-16	0.179	< 2.0 e-16	Except MTW-Car (p-value = 0.98), MThW-HV (p-value = 0.67) all are significantly different in mean speeds
	HV	100	40.54	40.37	8.12	0.81	-0.03	-0.09				
	MThW	1048	41.89	42.71	8.21	0.25	-0.85	1.91				
	MTW	3843	46.84	48.17	12.39	0.20	-0.61	0.96				
5:30 p.m. to 6:00 p.m. (Congestion)	Car	571	16.08	15.3	3.04	0.13	2.93	1.92	4.46e-03*	0.02332	4.287e-05	No significant difference in speeds except MThW-Car(p=0.0003) and MThW-MTW (p=0.0002)
	HV	133	15.91	15.3	2.46	0.21	0.68	-0.24				
	MThW	405	15.26	14.7	3.84	0.19	3.21	2.00				
	MTW	400	16.34	16.3	4.24	0.21	0.40	0.52				

** σ=Standard deviation, S.E.mean=standard error of the mean, e=10*



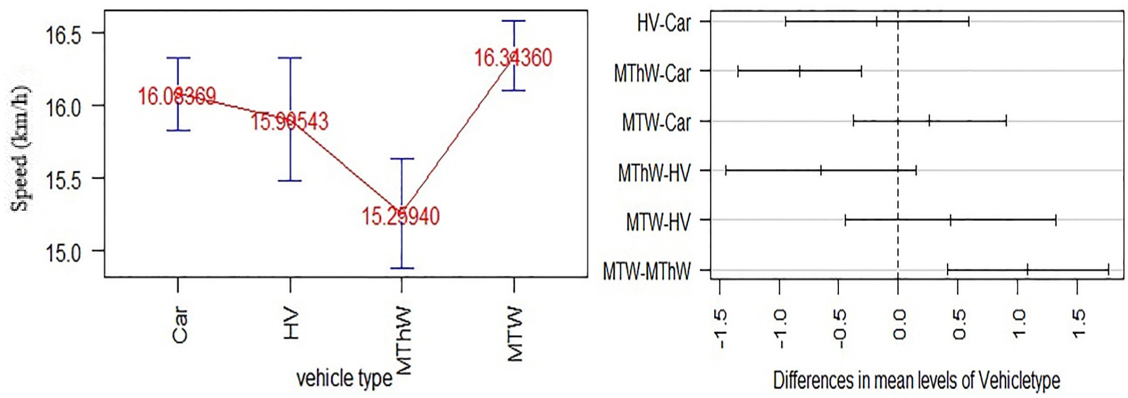
(a) (b)

Fig 7 (a) Mean plot with 95 % confidence interval (b) Tukey HSD95% family wise confidence level test at free flow condition[6]



(a) (b)

Fig 8 (a) Mean plot with 95 % confidence interval (b) Tukey HSD95% family wise confidence level test at capacity condition



(a) (b)

Fig 9 (a) Mean plot with 95 % confidence interval (b) Tukey HSD95% family wise confidence level test at congested condition

3.2 Impact of lane position on speed

To evaluate the effect of lane position on vehicle speeds the study assumed that the average vehicle speed and variances do not change across the width of the road. To confirm this, speed data between the lanes such as median lane (right most)and shoulder lane (left most) were investigated. However, center lane is not considered for the analysis (It is assumed that center lane acts like a via-media) and it is considered as a limitation of this study. For vehicles travelling in median and shoulder lane, F-test and two sample t –test[7] were chosen to test the homogeneity of variance and average speeds respectively. All the assumptions[8] of the tests are checked and validated. Results (Table 2) shows that data is satisfying the normality and homogeneity of variance assumptions. Test results (Table 2, Figure 10 and 11) confirmed that vehicles move faster in right lane than the left lane at free flow and capacity conditions. However, F-test results show that vehicles behave identically in these lanes which infers the similar distribution of driver classes. The difference in speeds probably arises from the type of vehicles present in the lane, merging and diverging manoeuvres. Equality in variance concludes that vehicle dynamics can be described by considering the whole width of the road as a single unit instead of separate entities.

Table 2: Statistical analysis for vehicles on different lane positions

Flow type	Vehicle types x*-y*	Mean of x	Mean of y	σ_x	σ_y	F-test p-value	t-statistic	t-test p-vale
Free flow	All-All	49.88	46.76	6.64	6.73	0.68	8.03	3.558e-15
	Car-Car	50.03	47.70	6.70	7.07	0.28	5.10	2.514e-7
	HV-HV	45.25[1]	44.24[1]	9.10	5.00	-	-	-
	MThW-MThW	47.97	45.23	4.92	5.80	0.05	2.10	0.02
	MTW-MTW	50.90	49.24	7.17	7.39	0.69	1.82	0.04
Capacity	All-All	39.75	37.70	7.43	6.80	0.05	4.68	1.7e-6
	Car-Car	39.28	38.56	7.63	7.54	0.85	1.10	0.14
	HV-HV	43.60[2]	34.40[2]	3.65	5.62	-	-	-
	MThW-MThW	38.80	36.56	6.30	5.28	0.08	2.49	0.01
	MTW-MTW	42.38	39.60	6.78	7.02	0.74	2.85	0.00
congestion	All-All	12.70	16.98	4.60	7.61	0.06	-6.72	0.00
	Car-Car	11.79	14.80	3.10	5.25	0.07	-4.04	0.00
	HV-HV	-[3]	13.86	-	6.12	-	-	-
	MThW-MThW	11.72	14.40	4.05	5.15	0.085	-1.95	0.03
	MTW-MTW	15.90	20.90	6.64	8.84	0.045	-3.57	0.00

*x-median lane, y-shoulder lane, σ= standard deviation,
e=10, [1,2]= sample sizes differ a lot (heavy vehicle rarely use median lane), [3]= heavy vehicles are not present

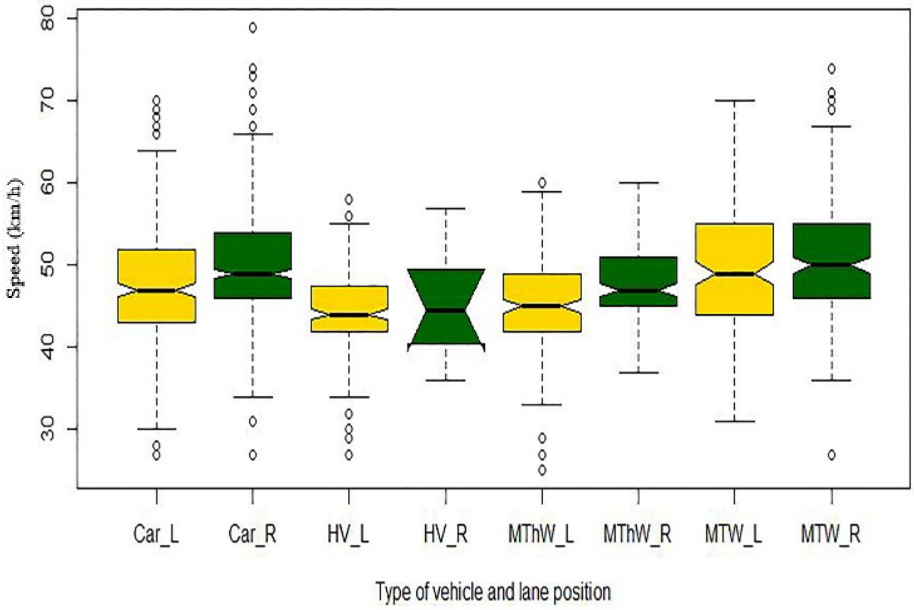


Figure 10: Distribution of vehicle speeds at free flow (L-left lane, R-right lane)

In congested conditions, vehicles in the left lane (close to shoulder) are maintaining higher speeds than the vehicles in the right lane (Figure 12). Cars only traffic is observed in the right most lanes during congestion period and they are maintaining strict lane discipline. It is also observed that the mean and variance of MTW's speed in the left lane are comparatively higher than another category of vehicles. It shows that lane effect needs to be considered while modelling congested traffic condition.

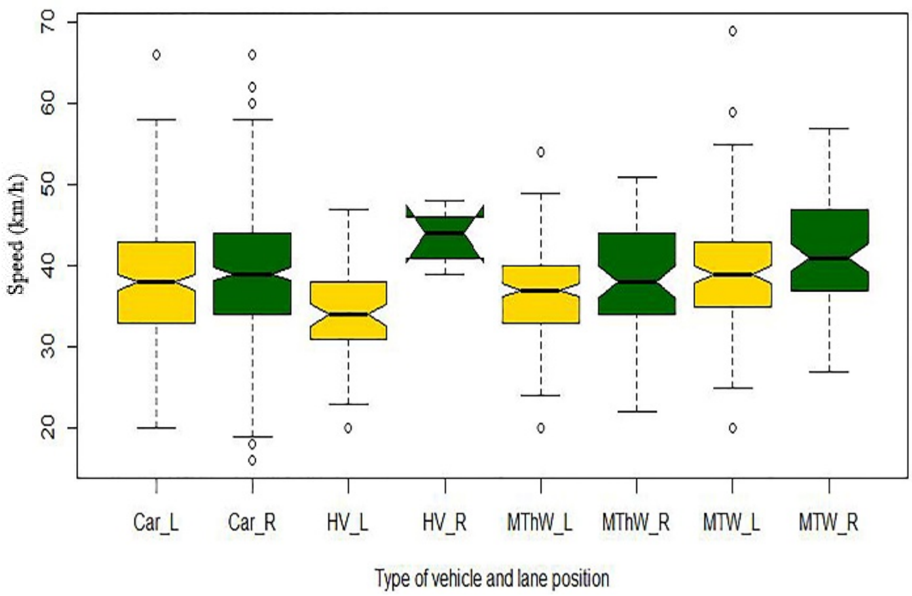
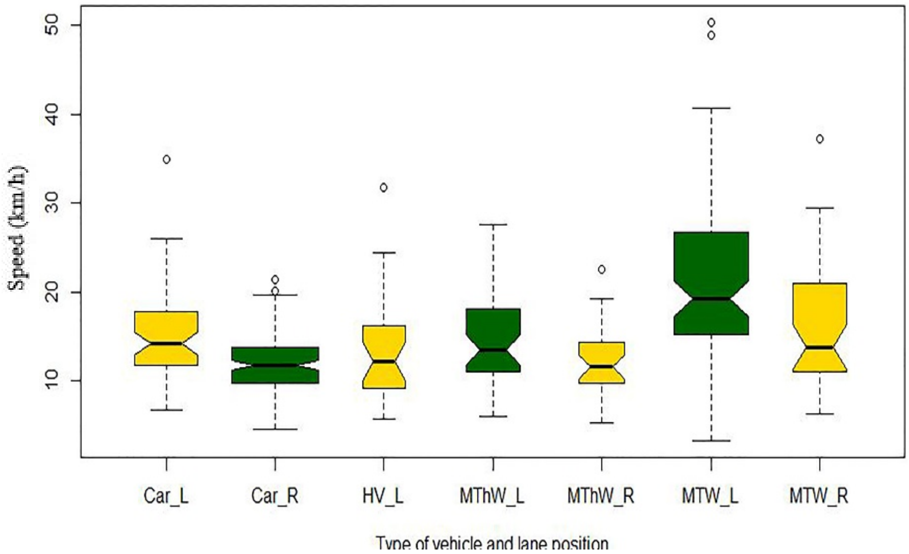


Figure 11: Distribution of vehicle speeds on different lane positions at Capacity (L-left lane, R-right lane)



(c)

Figure 12: Boxplots showing the distribution of vehicle speeds on different lane positions atCongestion (L-left lane, R-right lane)

4. Conclusions

Following inferences can be drawn from the present study:

1. Statistical analysis of difference between means of vehicular speeds shows that there is a clear distinction between different classes of vehicles in terms of speed maintained at different densities. It is clear that heterogeneous traffic streams on urban arterials are dominated by Motorized Two wheelers (MTW) and cars. However, except at free flow conditions average speeds of MTW are significantly higher than other vehicles. Due to the small size, they are able to seep through the gaps between vehicles and maintain higher speeds even at high density conditions.
2. Standard deviations (S.D) of different vehicular speeds reveals the presence of different driver classes and their adoption of speeds with respect to density on the road. In the present study, S.D's are very high for fast moving vehicles than the slow moving vehicles. Larger available headways and high acceleration capabilities encouraging fast moving vehicles adopting their own desired speeds whereas low acceleration capabilities were limiting the speed for slow moving vehicles.
3. In congested conditions, MThW are behaving differently from those of other vehicles in terms of adopting speeds. However, it is evident from the results that the number of driver classes are reduced with increasing density (reduction in variance with increasing density). It is clear that, modelling accuracy will be improved if we take different driver classes in to account. Especially scattering or non-stationary phenomenon in flow-density diagram can be explained.
4. Multiclass vehicle and driver modelling strategy may yield good results. Except at congestion, the variance between the lanes does not differ much.Therefore dynamics of the urban arterial cross section can be modelled in an overall manner.

Notes

[1]The speed data of different classes of vehicles are falling within 95% of confidence envelop except at tails. It is suggesting that the normality assumption is valid. If normality assumption failed select non-parametric tests such as Kruskal-Wallis H Test
[2]Positive and negative skewness values indicate the presence of outliers. Almost in all the cases, the data is nearly symmetric
[3]Kurtosis values revealed that the data in all the cases is nearly peaked
[4]The assumption of equal variance is more stringent than normality assumption in ANOVA. Unlike the normality assumption, equality of variance assumption is less sensitive to the large sample size and this assumption is more significant in unbalanced design (different sample size of the groups)
[5] p-value (Bonferonni p =0.025, 0.033) suggest that there are significant outliers in the data set
[6]If an interval does not contain zero then the corresponding means are significantly different
[7]Pooled or Welch t-test based on homogeneity of variance value. Alternative hypothesis used in this analysis is $\mu_1 > \mu_2$
[8]The assumptions are: samples from each population are random and independent, the population distribution of each sample must be nearly normal or the size of the sample is large

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