Fractal Assessment of Road Transport System

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Abstract

Transport system development is considered to be a key to rapid modernization. Urban transport system have less theoretical research, only some developed countries have carried out its evaluation and hence it has great potential for development and application prospects. Of the various alternate ways of characterising the spatial structure of transport system, deriving the fractal dimension forms a statistical method for measuring the space filling efficiency of the structure. However the studies using fractal geometry has limited to evaluating the level of fractality of either whole city or parts of cities. None of the earlier studies have tried to identify the variation in level of fractality within the city itself or tried to compare the fractal dimension between parts of the city with the whole city. The present study derived a simple statistical model to indicate the space filling efficiency of the transport system applying box dimension, considering Calicut city as the study area. The study indicates that urban road transport system of Calicut city has fractal dimension ranging from 1.004 to 1.542. This advocates that planar urban transport system fractal dimension (D) is $1 < D < 2$, with minimum value corresponding to an outer suburban zone and maximum corresponding a Central Business District zone. Again it reveals that higher fractal dimension relates to transport network with better connectivity and coverage of central district zones. The quantitative relationship identified between urban road transport system dimension and development pattern provides an empirical guide for planning and policy issues of urban development.

Keywords: Transport System, Box Dimension, Fractal.

1. Introduction

Transport system development is considered to be a key to rapid modernization, especially for better traffic efficiency. It exhibits a very close relation to the style of life, the location of activities and the goods and services which will be available for consumption. Urban transport system have less theoretical research. Only some

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developed countries have carried out urban transport system evaluation and hence it has great potential for development and application prospects. Alternate ways of characterising the spatial structure of transport system include evaluating the connectivity and density of the transport network. Yet another method of analysing the transport system is to obtain the centrality characteristic of the structure. Rather than the spatial structure of the transport system, the role of distinctive nodes and connections and their comparison with actual traffic flows are the primary interests of geographic inquiry. Still another method for evaluating the transport system structure is through the use of fractal geometry.

Being major structure of urban form, urban transport system shows fractal properties. Existing methods of characterising a system based on connectivity or density can only provide some result in certain scale. But fractal analysis evaluates a system from macro to micro and provides more quantitative spatial information. Fractal dimension is an important character of fractals that contains information about their geometrical structure at multiple scales. It allows to measure the spatial pattern by evaluating how fast the measurement increases or decreases as our scale becomes larger or smaller. Thus fractal dimension is a statistical magnitude measuring the space filling efficiency. The basis lies in the fact that fractal dimension increases with the space filling efficiency of the structure. However the studies using fractal geometry has limited to evaluating the level of fractality of either whole city or part of the cities. None of the earlier studies have tried to identify the variation in level of fractality with in the city itself or tried to compare the fractal dimension between parts of the city with the whole city. This study aims to derive a simple statistical model to indicate the space filling efficiency of the transport system in the city level. The box fractal dimension is an effective mathematical tool to describe a fractal structure. GIS provide a quick and easy way of monitoring and analysing the network. Hence this paper discusses the application of fractal analysis in transportation planning and attempted to characterise the spatial pattern of road transport system of Calicut city.

2. Objectives

1. To review the application of fractal geometry for characterising the spatial pattern of road transport system.
2. To assess the fractality of the road transport system within each zones of the corporation area of the city.
3. To obtain the fractality of the transport system of the corporation area as a whole so as to compare the pattern at two different scales, city level and zonal level.

3. Literature Review

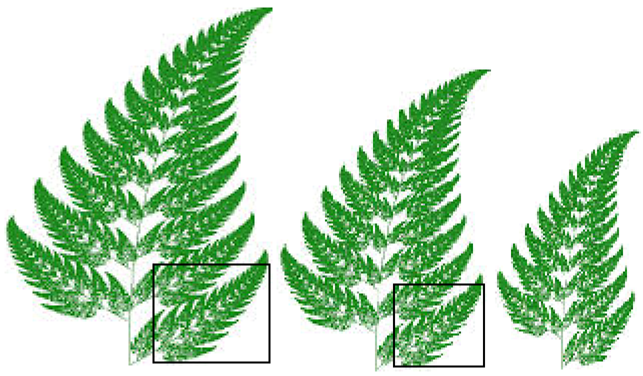
3.1 What is Fractal Analysis?

Fractal analysis is one of the significant quantitative methods of analysis of the spatial pattern of natural and social system. It is a contemporary method of applying nontraditional mathematics to patterns that defy understanding with traditional Euclidean concepts. It consists of methods to assign fractal dimension to quantify the fractal characteristic of a dataset or an object. Fractal analysis was developed by Benoit B. Mandelbrot (Mandelbrot, B. B., 1983) to characterise those patterns within nature that cannot be effectively quantified using classical geometry of whole-number dimensions.

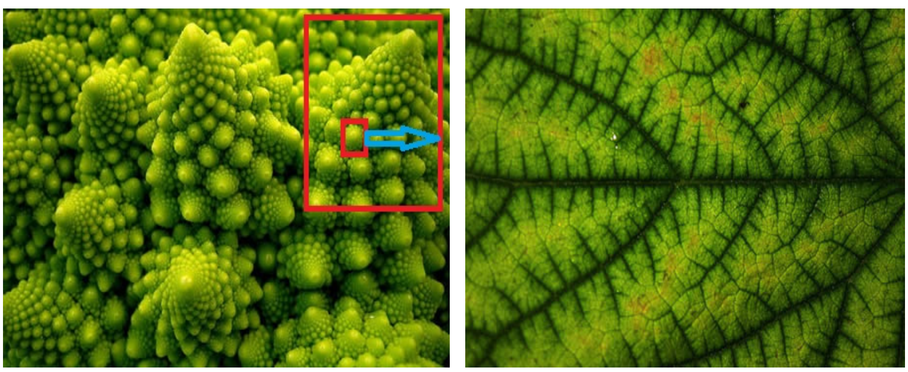
3.2 What are Fractals ?

Fractals are rough or fragmented geometric objects that can be split into parts, each of which is at least approximately a reduced copy of the whole. The term "fractal" was coined by Benoit Mandelbrot in 1975 and was derived from the Latin *fractus* meaning "broken" or "fractured." A fractal has following characteristics.

1. A fractal is self-similar. Parts of a fractal look like the whole, remain the similar form of irregularity from scale to scale.
2. A fractal is scale invariant. That is, appears to be the same at all scales of observations.
3. A fractal is not smooth and shows rugged everywhere, which means the fractal could not be exactly measured in Euclidean geometry.
4. A fractal possess to be infinitely complex. That means zooming in will bring up more and more details until infinity.



a) Scale- invariance fractality of leaf.



b) Self-similarity of fractal. c) Ruggedness and complexity of fractal.

Figure 1. Properties of fractals.

3.3 Types of Fractal

It is useful to make a distinction between two types of fractal objects, namely mathematically constructed fractals and natural fractals.

Mathematical fractals are mathematical constructs formed by the repetition of a simple geometric instruction. For example, a Sierpenski triangle as shown in Figure 2, or a Koch curve as shown in Figure 3.

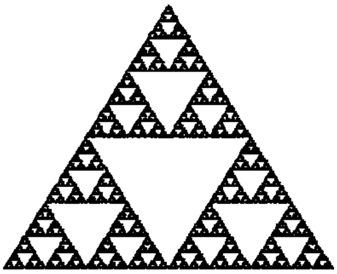


Figure 2. The Sierpenski triangle.

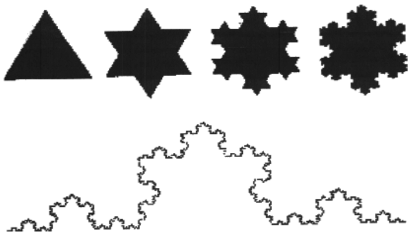


Figure 3. The Koch curve.

Natural fractals include the structure of mountain, lake, cloud, coastline, cauliflower, tree or airways in lungs. Natural fractal objects display a degree of randomness in their details that sets them apart from the mathematical constructs. Examples are displayed in figure 4 and 5.

Figure 4. Cauliflower.

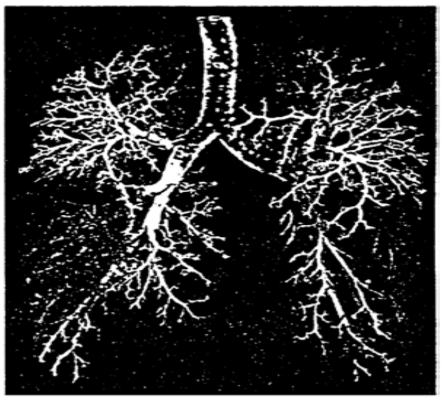


Figure 5. Airways in lungs.

In a pattern that demonstrates natural fractal characteristics the irregularity looks the same when the image is magnified, although the actual details may differ. In mathematical fractals, this similarity can be repeated over an infinite number of scales, while in natural fractals, it is repeated over a limited number of scales.

3.4 Fractal Dimension

The notion of fractal dimension is common to both types of fractal object. In fact, fractal dimension specifies how to relate a small part of something to the whole and can be defined as a value which is strictly greater than Euclidean dimension. The concept of fractal dimension enables the degree of irregularity of a shape or object to be calculated and represented as a number. This number lies between the Euclidean dimensions of 1, 2, or 3 as in figure 6. Essentially, fractal dimension is a measure of how well a particular object fills the space in which it is drawn. A series of points along a line has a fractal dimension between zero and one, because it occupies more space than a single point, but less than that of a line. Similarly, a squiggly line has a fractal dimension between one and two, because it occupies more space than a straight line but does not occupy that of a full plane. Likewise, a rough surface has a fractal dimension between two and three, because it occupies more space than a plane but does not occupy that of a full cube. The non-integer values exhibited by the fractal dimensions stem from the general scaling law, in which the quantity N of the object varies as a power of the length scale L. The exponent D is the fractal dimension of the object and k is a constant. Goodchild and Mark (1987) defined fractal dimension as the most important single parameter of an irregular cartographic feature, just as the arithmetic mean and other measures of central tendency are often used as the most characteristic parameters of a sample.

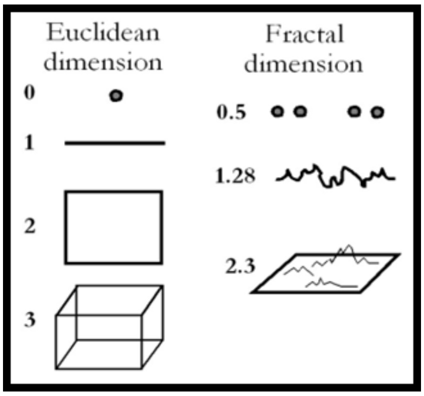


Figure 6. Variation of fractal dimension. (Source: Zmeskal et. al, 2008)

3.5 Types and Methods of Measuring Fractal Indices

There are a variety of fractal dimensions, including Hausdorff-Besicovitch dimension, Minkowski-Boulingand dimension, box counting dimension, the capacity dimension, the similarity dimension, the correlation fractal dimension etc.. Fractal dimension of transport network can be determined in number of ways as follows.

1. Structured walk/ divider method suggested by Kaye (Batty and Longley, 1994; Batty and Xie, 1996) and modified by Kent and Wong (1982), Kaye and Clark (1985).
2. Equipaced polygon method suggested by Kaye (Batty and Longley, 1994; Batty and Xie, 1996) and elaborated by Kaye and Clark (1985) is based on the perimeter lengths computed for the sequence of scale changes.
3. Cell count method suggested by Goodchild (1987), Morse et al. (1985).
4. Mass-radius method is based on the image portion found within a set of concentric rings covering the image (Kaye, 1978; Kaye, 1989).
5. Hybrid walk method suggested by Clark (1986) is based upon the geometric chord length series as the structured walk method.

6. Calliper method is based on linear measurement sizes and steps.
7. Pixel-dilation method calculates the Minkowski-Boulingand dimension based on a set of infinitive small circles.
8. Box-counting method has been advocated in many studies (Kent and Wong, 1982), and is used for urban transportation fractal analysis.

Each of these methods can be used to analyse spatial objects ranging from strictly self-similar to non-self-similar for a range of scales.

3.6 Application of Fractal Analysis in Transportation Planning

A city has very complex subsystems such as population, land use and transport system. Transport network and other subsystems in a city show the properties of fractal. Therefore fractal analysis form an effective tool for their quantitative evaluation and representation. Previous researches did some analyses between cities or discrete analyses in one city. The use of fractal dimension in urban analysis has already been carried out by a number of authors. As to certain subsystems of the city many researches are available, such as land-use patterns (Liu et al., 2012; Chen and Jiang, 2016), population distributions (Longley and Batty, 1989), or transportation networks (Mandelbrot, 1983; Morse et al. 1985; Peitgen and Saupe, 1985; Peitgen et al. 1992; Rodin and Rodina 2000; Feng et al. 2008). Fractal analyses of urban transportation networks generally fall into two groups, those aimed at revealing the regularity and self similarity of road systems (Mandelbrot, 1983; Peitgen et al. 1992; Feng et al. 2008) and those that go one step further by trying to link the fractal properties of transportation networks with cities' properties and functions (Morse et al. 1985; Peitgen and Saupe, 1985).

4. Study Area and Data

The area selected for this study is Corporation area of Calicut district in Kerala state. Calicut city is situated in the South-West corner of the district. Calicut city extends between 75º 47' 23" E and 76º 26' 40" E longitudes and 11º 30' 08" N and 11º 58' 40" N latitudes. Calicut city corporation area with 75 wards, has an area of 84.29 square kilometer and has a population of 4,32,097 as per 2011 data with gross density of about 5126 persons per sq.km. Figure 7 gives the zone map of the Calicut Corporation.

The total length of the road network within the urban area is about 335 kilometers. The road density is 3.39 kilometer per square kilometer. The roads in the entire network can be broadly divided into categories based on their functions, viz., the roads facilitating intercity trips and roads facilitating intra city trips. National Highways and State Highways are the major roads facilitating intercity trips. The city roads, which connect the residential neighbours with the major road or with the CBD and other work centres, are meant for intra city movements. Figure 8 shows the digitised road network map of Calicut Corporation.

Figure 7. Zone map of Calicut city. Figure 8. Road map of Calicut city.

5. Data Collection and Database Preparation

The base map of the road network based on Calicut Corporation, was obtained from Google Maps. AutoCAD drawing of the wards in the corporation area of Calicut were obtained from the Calicut Corporation office. The methodology of this study involves application of GIS software for evaluating the fractal characteristics of the road transport network of the study area. ArcGIS 9.3 was used for characterising the network based on spatial pattern and hence to identify the variation in their pattern. All the roads including Arterial, Sub-arterial, Collector Streets and Local Streets were digitised from the Google base map. The Autocad ward map was converted into ESRI shape file format. Both the maps were geo-rectified in ArcGIS, for which the ground control points were used. In ArcGIS, the ward boundary and roads were converted into polygon and polyline features respectively. The layer of ward boundaries were superimposed on the layer of Calicut city road network to extract the road network within each ward and hence the fractal dimension of each road network pieces were evaluated for studying the spatial pattern variation and hence the performance of the network. In this study, the principle of box dimension has been utilised throughout to attain the fractal dimension.

5.1 Methodology of Box Dimension Computation

Each of the zonal networks, prepared in scale 1:10000 was superimposed by square grids of unit box size 1000 km as in Figure 9.

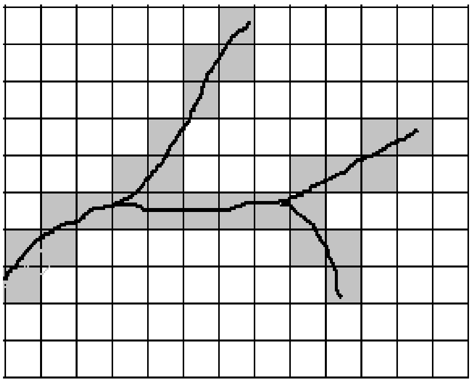


Figure 9. Schematic illustration of box-counting method to obtain fractal dimension of road transport system.

Number of small unit boxes which intersect the network plane was counted, which represents the box count N(s) corresponding to cell size ‘s’. The cell size of unit square was reduced alternately. Simultaneously the box count was obtained and prepared their natural logarithmic values ln (N(s)). To compare the results of measurement at different scales and subsequently to calculate the fractal dimension, the measurements were entered into a double logarithmic graph as the natural log of N(s) against the natural log of s (the grid size), where N(s) is the resultant lengths. This helps to remove the difficulty of reading length-versus settings relationships when the settings used may vary from several hundred units to just a few. These log/log diagrams are referred to as Richardson plots, after Richardson (1961). When points on a log/log diagram fall on a straight line, a power-law relationship exists between the two sets of data (Peitgen et al., 1992). This allows the exponent of that power law to be read off as the slope of that straight line, D. To arrive at the value of D, the equation y = Dx + b was to be employed, which forms the description of a straight line on an x, y diagram, where b is the intercept point of the straight line on the y axis and d is the gradient of the line. So D = (y_2-y_1) / (x_2-x_1) for any pair of points, for example (x_1 , y_1) and (x_2 , y_2) on a line, which can be calculated easily by picking two points on the line and their co-ordinates and subjecting them to the equation. The value of this is in effect the gradient of the line. This value ‘D’ is essential in calculating the fractal dimension of the subject in question.

Following this procedure the road network within each of the zones of the study area were evaluated using the box counting fractal geometry.

Advantages of box fractal dimension

- It is relatively easy and effective for empirical estimation.
- It is one of the most commonly used method.
- This method may be applied to non-self-similar sets.

Disadvantages of box fractal dimension

- Changing the orientation of the box grid changes the box count, which result in a variation in the value of the fractal dimension.
- The value of fractal dimension will be slightly influenced by the resolution of the grid.

- Box counting estimation is difficult if one operates on data other than one or two dimensional data.

6. Analysis and Results

6.1 Fractal Dimension Computation using Arc GIS

Mesh of grid size 1 km was prepared in ArcGIS. Then the road network digital map within each zone was overlaid by this mesh in ArcGIS and obtained the count of the boxes required for covering the road network completely, from the corresponding attribute table. This process was repeated for various reduced grid sizes. Then a ln-ln plot of the cell size (x-axis) and the box count (y-axis) was prepared as a linear plot. Regression slope of this fitted straight line indicates box-counting fractal dimension.

6.2 Estimation of Fractality of Transport Network within Zones

The zone-wise road network data extracted by clipping the Corporation boundary of each zone on the road transport network as in figure 10 occupying ArcGIS software, was quantified in terms of box fractal dimension. Figure 11 shows the square grid prepared for overlaying the transport network.

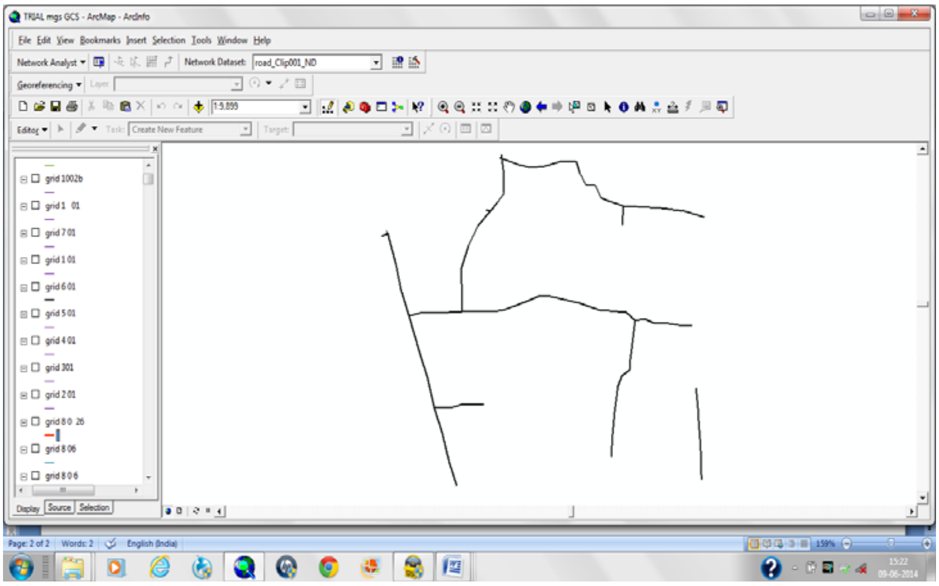


Figure 10. Road network data clipped using zone boundary.

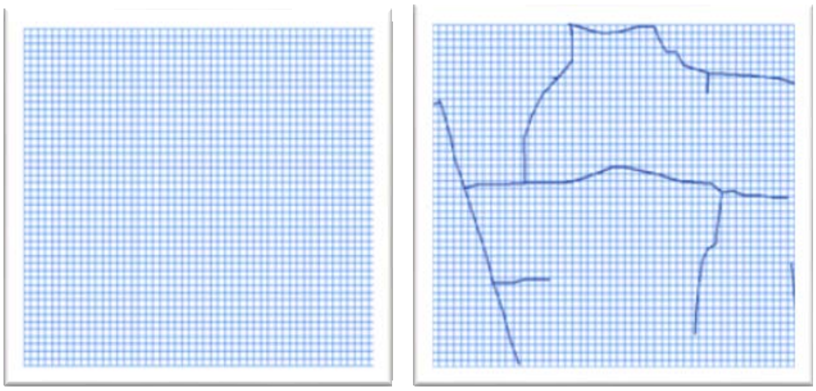


Figure 11. Grid overlaid on the road network.

Overlaying the grid on the road network, the number of cells overlapped by road network were obtained in ArcGIS. Changing the cell size the same procedure was repeated and obtained ln-ln plot of Count Vs Grid size as in figure 12. Slope of this linear plot give fractal dimension.

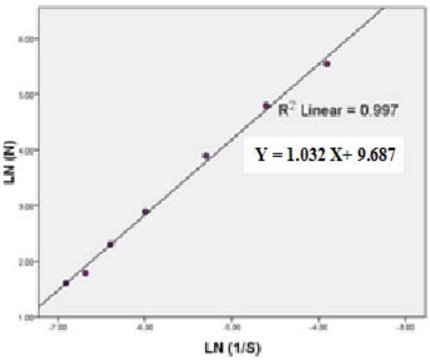


Figure 12. Logarithmic plot of count and grid size.

Following this procedure, fractal dimension of all the 75 zones were obtained, values of which varied from 1.004 to 1.542, with mean value of 1.188 and standard deviation 0.138. The minimum value 1.004 corresponds to Chettikulam which is a suburban zone. The maximum 1.542 corresponds to Kuttichira, which is a zone near the Central District. Visual thematic analysis was performed to obtain the variation of fractal dimension as shown in figure 13.

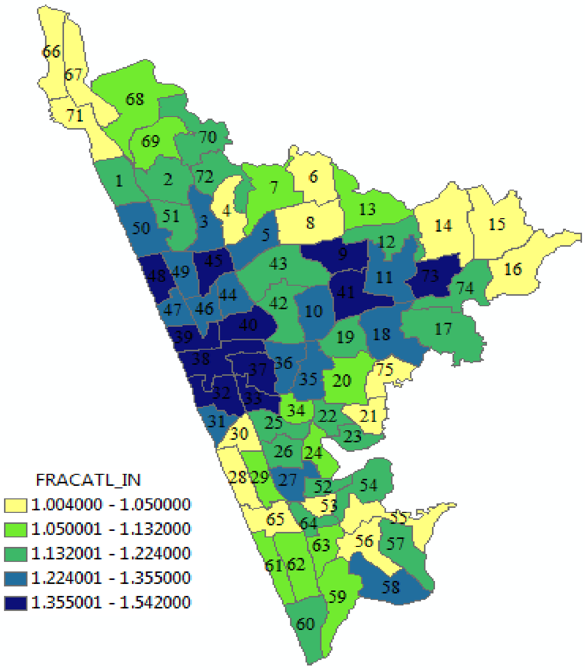


Figure 13. Thematic map showing variation of fractal dimension.

The inner central tracts have dimension approximating 1.54. The dimension of outer suburban tracts approximates to 1.0, while the central tracts have values in between.

6.3 Fractality of Transport Network of Whole City

To compare the spatial pattern of the transport system of the city as a whole with that of each zone, box-counting method was applied to measure the fractal property of the urban transport system of the whole city as in the case of each zone. For this, grids of progressively smaller sizes were overlaid on the transport system within the entire city and simultaneously obtained the box count were the system was present. Double logarithmic linear plot of count and cell size was prepared, whose gradient was obtained as 1.480 which corresponds to the fractal dimension of the city transport system. This means, the road network pattern as a whole indicates a fractality in between that of the central CBD zone and the outer suburban zones.

7. Conclusion

Fractal geometry can also provide a synthetic measurement of the complexity and irregularity of the phenomena there by allowing a numerical characterization. Fractal analysis will be able to precisely characterise the spatial pattern of the city and its subsystems providing unique numerical values that form an index of roughness of the geometry pattern. The fractal dimension of a pattern varies depending on the degree of roughness and ruggedness of the pattern, and will have a different value for each pattern type, with the fractal dimension being specific for that pattern.

This study applied box-counting method to describe the fractal property of urban transport system. With a case study of Calicut city, the analysis of the whole urban area as well as each of the urban zonal wards were carried out based on the box fractal dimension, which observed a logarithmic correlation between the changing grid sizes and the changing numbers of the researching objects within their corresponding areas. In terms of the detection results, it can be discovered that the dimension of the central zonal tract (D_1) is bigger than the dimension of the outer suburban zonal tract (D_3), but smaller than the dimension of the inner suburban tract (D_2). Further, $D_2 > D$ (the dimension of the whole urban area) $> D_1$ is another research finding.

It is a fact that the fractal measure is very much different from other measures of evaluation of transport system. This is because fractal dimension can extract not only the whole feature of the network, but also the system inner pattern and its gradual change in space, while other measures such as connectivity are unable to explain the system inner pattern. The relationship between parts and whole in city planning and urban transport planning can be explained to some extent through the detection and comparison of box fractal dimensions. Evaluating the urban road transport system applying fractal theory in GIS not only improves the precision of the quantitative index for describing transport system spatial pattern, but also offers a strong tool for studying urban transport in multi level. It can be seen that the box fractal dimension reflect the spatial pattern of the transport system filling the urban space.

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