



# MINLP MODEL FOR OPTIMUM TRAFFIC COUNTING LOCATION FOR ORIGIN-DESTINATION MATRIX CORRECTION

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## Abstract

Trip demand information as trip OD matrix has important rule in transportation planning & management, and performing a basic role in transportation systems studies. Access to this matrix with general method requires so much time, large manpower and much cost. In recent years many research for access to OD matrix with traffic count information in all of network links or some of them are done. Validity of Corrected OD matrix related to input data and the most important input are number of traffic count and locations of them. In this paper proposing a MINLP locating traffic counts model for OD matrix correction, and solve this model for a small network with genetic algorithm.

*Keywords: ODmatrix, transportation, MINLP, traffic count.*

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## 1. Introduction

Trip demand information as Trip OD Matrix is one of the most important and main entries in transportation planning engineering and is considered one of the basic information for designing and management of transportation systems. Access to these information is very difficult, time-consuming, cost-consuming and also manpower-consuming. Generally, trip OD matrix may be estimated by two methods of direct or using mathematical models. The method of using model includes two groups of using demand models or correcting trip OD matrix using traffic volume in all or some of the network links.

In the second method, using the models which obtain trip table based on traffic volume in the links, in some models, all network links and in some models, some other network links are used. It would be meant that some of the network links are selected and traffic volume are counted and then using these information, the demand matrix shall be estimated or corrected.

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To solve trip OD matrix estimation based on traffic volume in the network links, the numerous algorithms are used which the efficiency of some of them is proved in the great networks and they may use for solving the real problems, Such as Gradient Method Spiess (1990). Gradient Method is a iteration method which starts from the initial trip OD matrix and in any repetition, it tries to reach the matrix producing the currents near to the

observed currents in the network links and by changes ,this method is amended in such a way that the corrected matrix is remained near to the initial matrix. Inthis paper,a nonlinear-integerprogram for location of traffic count for correction of OD matrix is presented and solved it by genetic algorithm.

## 2. Literature Review

### 2.1. OD matrix correction methods based on traffic counts

First of all we proceed to gradient method Spiess (1990). This model was applied to minimize a mathematical convex function for large networks. The model has been used in EMME/2 transportation planning software. The objective function in this problem is a convex function that equal with the least square between the observed and estimated traffic link volume. This objective function can be described as follows.

$$\text{Min } F(T) = \sum_{a \in A} (v_a - \tilde{v}_a)^2 \quad (1)$$

S.t

$$V = P(T)$$

Where  $v_a$  is estimated traffic volumes and  $\tilde{v}_a$  is observed traffic volume on link a. The Spiess model can estimate non-zero values for OD pairs with zero trips considered initially, which can be a matter of concern for the planners who wants to preserve the structure of the estimated OD matrix. Thus, Doblas and Benitez (2005) further developed the Spiess (1990) model study for cases when one require to retain the structure of the estimated OD matrix. Lundgren and Peterson (2008) developed a heuristic bi-level problem and solving it by a descent algorithm and applied the algorithm for a large scale network for the city of Stockholm. Baek et al (2004) applied multiple-vehicle data for OD matrix estimation from traffic counts. The multi-vehicle OD matrix estimation method is given as:

$$\text{Min } F(t_w) = \frac{1}{2} \sum_c \sum_{a \in A} (v_a^c - \tilde{v}_a^c)^2 + \frac{\gamma}{2} \sum_c \sum_{w \in W} (t_w^c - \tilde{t}_w^c)^2 \quad (2)$$

S.t.

$$T \geq 0$$

$$V = P(T)$$

where T is the OD matrix with trips elements for the vehicle type c between OD pairs w,  $\tilde{t}_w^c$  is the corrected OD trips of vehicle type c between OD pairs w, V as the vector with elements  $v_a^c$  as traffic volume of the vehicle type c on the link a,  $\tilde{v}_a^c$  is the observed volumes of vehicle type c on the link a,  $\gamma$  is the parameter reflecting the reliability of the corrected OD matrix and P(T) is the multi-vehicle traffic assignment matrix. For solving the non-convex problem, genetic algorithm has been used. The algorithm has been demonstrated only using a small network.

Cascetta and Posterino (2001) and Yang et al (2001) solved the OD matrix estimation problem including congestion effects by considering different traffic assignment models. Also there is another procedure for estimating OD matrix based on statistical

approaches. Hazelton(2008),Van Aerde et al(2003) and Li(2005) do more studied via this procedure.

## *2.2.Traffic Counting Location*

The first research performed in the field of traffic counting links locating are the studies by Yang and et al (1991). In this study, they examined the importance of the place of volume counting links in the estimated trip OD matrix validity and presented some methods for volume counting links locating. ‘

In other research, Yong et al (1998) used the factors of maximum possible relative errors to examine trip OD matrix validity obtained from the volume of traffic counting links information and presented some rules for locating volume counting points which these rules are based on selection of location with the most traffic volume, participating in the most active paths and the independency to each other.

Yim et al (1999) proposing a new method for locating the traffic counting links which is based on the two following rules:

Rule 1: In a network, the best set of volume counting links is a set that the most trip pass through them.

Rule 2: While most of the sets are the equal from passing net trip point of view, the set which has the most total trips, is most important in this regard.

They determined a mathematical model for traffic counting links locating based on these rules and also proposing an innovative algorithm for solving this model which its solution is possible for the large scale network.

Hyung and Chang (2003) also studied to select the optimum location of the traffic count links based on minimize traffic volume count. Bianco et al (2001) proposed heuristic algorithm and solved the sensor location problem by defining a two-stage method; determining the minimum number and location of counting points with known turning probabilities (assumed) and estimating the OD matrix with the resulting traffic flows.

Yang et al (2003) studied the planning installation of traffic counting detectors for long duration and solve it by Genetic Algorithm. Kim et al (2003) presented two models, link-based and road-based model, to determine the location of the traffic count links which minimizes the total cost. Ehlert et al (2006) optimized the additional counting locations assuming that some detectors are previously installed expanding the OD coverage.

Gan et al (2005) studied both the traffic count detector location and the error estimation measure in OD matrix estimation problem taking into consideration the route assignment models and the levels of traffic congestion on the networks.

Yang et al (2006) formulated an integer linear programming (ILP) problem using shortest path-based column generation procedure and branch-and-bound technique in determining screen line based traffic counting locations.

### 3. MINLP model for traffic counting links location

Transportation network  $G(N,A)$  is assumed which  $G$  indicates network,  $N$  indicates nodes and  $A$  indicates the links set of this network. Assuming  $g^0$  is the initial OD matrix, using traffic assignment problem, this matrix is assigned to the network and the traffic volume of all network links are obtained. Set of traffic volumes of network links is displayed by  $V^0$ .  $g^0$  matrix components are changed randomly to create a new matrix such as  $\bar{g}$ . Its manner is as follows that each matrix components is randomly changed in such a way that increases or decreases for  $\alpha\%$  in compare with each of its analogous component in  $g^0$  matrix.

It would be meant that:

$$(1 - \frac{\alpha}{100})g_{OD}^0 \leq \bar{g}_{OD} \leq (1 + \frac{\alpha}{100})g_{OD}^0 \quad OD = 1, 2, \dots, S(3)$$

$S$  is the number of OD matrix members. The ruined matrix is obtained that its component has been changed at most  $\alpha\%$  from up and down randomly in compare with  $g^0$  matrix. This matrix is assigned to the network and the created traffic volume is displayed with  $\bar{V}$  set in the links. Presently using gradient algorithm and  $V^0$  traffic volume in some of the network links, as the observed links,  $\bar{g}$  matrix is corrected until  $\hat{g}$  matrix is produced. Then by correlative coefficient, we compare the  $\hat{g}$  corrected matrix and  $g^0$  initial matrix.

Although selection of the links which participate in many active paths of the network or has the highest traffic volume, initially an appropriate strategy is for choosing volume counting links, but two factors may guide to more appropriate selection. First, surely, selection of a link in the active paths between OD pair may assist the demand correction of its OD pair. Additionally, in case, the number of links selected in active paths between OD pair, it shall be more effective. But necessarily, this effect is not appropriate with the selected links linearly. On the other hand, the second selection of the link has not the effect of the first link in the active paths between OD pair and its severity shall be decreased by links increase. Second, based on the numerous numbers of Os-Ds, it may be impossible to choose the links in such a way that for each OD pair, one of the links should be determined in active paths between its OD between the selected links. Therefore, in selection of volume counting links, it is better to use the links that participates in active paths between Os-Ds by more demand.

In this paper, for consideration of two factors mentioned above, the following mathematical model is presented for locating traffic counting links.

$$Max \sum_{OD \in S} \frac{g_{OD}^0}{X_{OD}^2} \quad (4)$$

S. t.

$$X_{OD} = \sum_{a \in A} (\sum_{k \in K_{OD}} \delta_{ak}) y_a \quad \forall OD \in S$$

$$\sum_{a \in A} y_a = M$$

$$y_a = 0 \text{ or } 1 \quad \forall a \in A$$

$$X_{OD} > 0 \quad \forall OD \in S$$

Where,

$A$ : network links set

$S$ : OD pairs set

$K$ : Active paths set in network

$K_{OD}$ : Active paths set for OD pair

$$\delta_{ak} = \begin{cases} 1 & \text{if link } a \text{ is in path } k \\ 0 & \text{if link } a \text{ is not in path } k \end{cases}$$

$g_{OD}^0$ : OD pair initial demand

$X_{OD}$ : number of selected pairs as the observed links between O-D pair

$M$ : number of links which shall be selected

$$a = \begin{cases} 1 & \text{if link } a \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

In locating model (4),  $\frac{1}{x^2_{OD}}$  is a function which determines the value of number of links selected in active paths between OD pair. This function is defined in such a way that in case two links is selected for one OD pair, the value of these links is not twice value of selection of one link and therefore, the existence of this function in the objective function causes that the number of links between one OD pair shall not be decreased uselessly.

Model (4) is a model of integer and non-linear programming which solving is impossible by the software available in real networks. Therefore, to solve this locating model, Genetic algorithm has been used.

#### 4. Results of model solving by Genetic algorithm method

For solving this model a small network with 75 links, 46 nodes and 196 OD pairs (with positive demand) has considered. For this network the initial OD matrix used in this paper,  $g^0$ , is an made matrix related to small network.

Initially, the traffic volume on all links of network is obtained from assignment of  $g^0$  matrix to the network. Using the results of this assignment, all active paths of network are determined too. Then  $g^0$  matrix shall be corrupted in such a way that all its components shall be changed randomly until 40% and thus  $\bar{g}$  matrix is produced. It would be meant that each  $\bar{g}$  matrix element is defined for the following equation:

$$0.6g_{OD}^0 \leq \bar{g}_{OD} \leq 1.4g_{OD}^0 \quad (5)$$

$$OD = 1, 2, 3, \dots, S$$

With selecting some of real links of the network, using mathematical model (4) and traffic flow from  $g^0$  matrix assignment to network as observed flows,  $\bar{g}$  matrix is corrected by gradient method and the corrected matrix is named  $\hat{g}$ .

This problem solved with the various links and its results are mentioned in the table (1).

As observed in table (1), by increasing number of links from 120 correlation coefficient link between  $g^0$  matrix and  $\hat{g}$  matrix, no noticeable growth is observed and since links volume counting is cost-consuming, this model is able to present appropriate result for correction of OD matrix, with selecting approximately 50% of the network links. It is possible to get better results for large scale network.

Table 1 Correlation coefficient between  $g^0$  initial matrix and  $\hat{g}$  corrected matrix

Number of links	Correlation coefficient
2	0.09
4	0.13
8	0.26
15	0.37
25	0.53
40	0.79
60	0.89
75	0.92

## 5. Conclusion

OD matrixes are one of the transportation planning and engineering elements and are interfered in most issues related to transportation. Access to these matrixes through direct methods are impossible because of being time-consuming. Therefore, some methods have been existed for correction of these matrixes which are performed using OD matrix and also traffic volume counting in some of the network links.

Quality and confidence level the estimated OD matrix is severely depend on the authenticity and accuracy of model entry data (accuracy of observed traffic volume and initial OD matrix and observed links location). In this paper, a locating model is determining for traffic count location and by solving this mathematical model by the genetic algorithms for sample small network, traffic count locations is obtained in this network. The proposed MINLP model is a new method for traffic count location for correcting OD matrix. At the end, this model is implemented for a small network and the obtained results in this research indicates the good efficiency of the mathematical model.

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