Analytical Modeling of Two-Level Urban Rail Transit Station Elevator system as Phase-Type Bulk Service Queuing System

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Abstract
This paper analyze the peformance of Urban Rail Transit Station (URTS) elevator service facility as Phase-Type (PH) bulk service queuing system $PH/PH^{[K]}/1/inf$ .The elevator system of URTS is a kind of unique bulk service queuing system of which arrival interval and service time distribution are of Phase-type, the queue is individual, having single server (Elevator car) and of infinite capacity. Each time the elevator process specific number of passengers in the form of batch. The Quasi-Birth-Death (QBD) process programmed in MATLAB is used for computation of stationary probabilities and performance measure. The performance analysis of elevator batch service queuing system including mean number of passenger in the system, mean waiting time of passengers in the system, mean queue length and mean time spent in the queue for elevator service facility are obtained. The results of $PH/PH^{[K]}/1/inf$ are then compared with existing batch servive queuing models i.e., $M/M^{[K]}/1/inf$ and $D/D^{[K]}/1/inf$ for the assessment purpose. The output indicates that other queuing models always underestimate the performance measures that results in improper design of facilities at subway station. Thus this research provides a more realistic fitting distribution for passenger arrival interval distribution and service time distribution of elevator service facility at URTS.
Keywords: Passengers, Bulk Service Queuing System; Phase-Type Distribution; Elevator Service Facility; Urban Rail Transit Station, Quasi-Birth-Death Process.

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1. Introduction

In Urban Rail Transit Stations (URTS), the passenger queues form a frustrating, yet vital part of our everyday lives. Queuing occur usually in case when there are insufficient resources. The service facilities at URTS are often of finite capacity which means that there is limited space available or limited number of facilities for passengers to get services from the service facilities. The performance analysis of these service facilities may tell us something about the mean time that the facilities will be in use, or the mean time that a passenger have to wait. The passengers queue formation during the peak periods results in the loss of service facilities performance. Therefore; information of such important performance parameters can help the planners and designers to make intelligent decision regarding the demand and capacity as well as when and how to upgrade the service facilities.

The passengers flow at URTS may be regarded as the users and the service facilities viewed as the resources. The service facilities and passenger flows encompass a type of unique queuing system with arrival process distributions and service time distribution that are diversified and not fixed. The past literature (Fruin 1971) and the existing codes (Group 2013; Shi et al. 2003) for the planning and design of URTS service facilities considered the passenger arrival process (or arrival rate to service facility) and service time (time spent in getting services) of service facilities as deterministic. They neglected the variation in passenger arrival interval (demand variation) as well as the randomness in the service time. The first thing to note in this context is that "URTS service facilities'' is a broad category. The URTS consist of many service facilities including elevators, Ticket Vending Machines (TVM), stairways, corridors, escalators, platform and etc. They all combine to form a kind of network. The number and sizes of these service facilities effect the walking time of passengers, the operation of other service facilities and the level of service (LOS). To narrow the scope of study, our paper will cover the performance analysis of elevator service facility at URTS only. A very limited details regarding the elevator service facility is provided by the existing design codes for the planning and design of transit stations (group 2013; shi et al. 2003). Also there are few shortcomings in the analysis procedures of service facilities based on existing codes. Thus, in order to offset the shortcomings, the authors describe the elevator service facility and passengers flow at URTS as $\mathrm{PH/PH^{K}/1/inf}$ queuing system with arrival process of passenger and service mechanism of elevator service facility follow Phase-type (PH) distribution.

The queuing theory concept has two different technique for performance evaluation of system i.e., analytical method by using mathematical models and Discrete Event Simulation method. The analytical queuing models have been used in the past in the various fields to apprehend the performance of systems. The queuing models have a special significance in modeling communication networks, manufacturing facilities, and traffic and transportation planning. In the past, Cheah and Smith (1994) presented M/G/C/C state dependent queuing model using the inverse linear and exponential velocity–density relationships to get the expected number and waiting time of people in the system. In the context of traffic and transportation, Vandaele et al. (2000) formulated some basic models using M/M/1, M/G/1, and G/G/1 frameworks to effectively represent the vehicular traffic flow process. In order to study passenger arrival and service processes at URTS, Jiang et al. (2010) established M/G/1 queuing model based on the exponential arrival interval and a general random service time for

URTS during the peak hours. They proposed a computation method for URTS considering the LOS.Similarly; (Løvås 1994) evaluated the passenger queuing behavior at URTS during emergency based on M/G/1 queuing network model.Jiang et al. (2015) established a G/G (n)/C/C state-dependent simulation model for the optimization of metro station corridor width considering the Level of Service (LOS). They used gamma distribution to fit the arrival process of passengers and log-normal distribution to fit the service process at corridor.Ying and Xing-fang (2008) used the multi-server M/M/C queuing model for the analysis and optimization of TVMSat URTS based on Poisson's arrival process and exponential service process. Jiang and Lin (2013); used G/G/C simulation model for evaluation and optimize the TVM at URTS with log-normal distribution for fitting arrival interval and gamma distribution for fitting TVM service time.

Most of the researchers in the past conducted their studies on elevators systems in high-rise buildings. They had focused on the up-peak pattern due to its critical nature in which passengers randomly arrive to the lobby and randomly select their destination floor. (Bailey 1954) for the first time presented a steady-state solution for the up-peak pattern of elevator system by using $M/Er^{[K]}/1$ queuing theory where arrival process is based on Poisson's distribution (M), Service process is bulk service i.e., customers are served in batches with K customers in each batch based on Erlang distribution (Er) and a single elevator as server. Zong et al. (2004)and (Xueqin 2006) analyzed the elevator system by using exponential distribution for fitting service process. (Chen and Jiang 2010) presented a discrete-time elevator queuing system for two-level ware house considering the down-peak pattern with general service distribution. (Siikonen 1997) analyzed the elevator with round-trip time as service process that follows the Erlang distribution. Lee et al. (2009)derived the probability distribution functions for queue length of passengers in the lobby, for the round-trip time of elevator considering up-peak pattern and for waiting, ride and journey time of passengers.

From the review of advanced stochastic processes, the authors got that PH distribution has substituted the exponential distribution in health care, queuing systems, manufacturing processes and optimization of communication systems. The reason to use PH distribution for fitting the arrival interval and service time is that it owns good analyticity, universality, and computability as theoretically it can be fit to any positive random number infinitely. It resulted in the emergence of ample PH-based queuing models, including PH/PH/1 by Krishnamoorthy et al. (2008) and PH/PH/1/C by (Alfa and Zhao 2000). In the transportation domain, Jiang et al. (2013) for the first time applied the PH distribution to fit the passenger flow arrival interval distribution in URTS that has revealed a good data fitting effect. That research opened the way for using PH distribution in the field of traffic and transportation. Recently; Reijsbergen et al. (2015) proposed a methodology of constructing stochastic performance model for public transportation network using PH distribution. Similarly; Hu et al. (2015) presented a PH/PH (n)/C/C state-dependent queuing model for the analysis and design of URTS corridors. None of the researchers have so far considered the use of PH distribution in performance analysis of elevator service facility at URTS. Therefore; this research aims to analyze the elevator service facility at URTS by using $PH/PH^{[K]}/1/inf$ queuing model due to the fact that there is randomness in the arrival process of passengers to the entrance of elevator and randomness in the service process of elevator. The superscript 'K' over the service process in $PH/PH^{[K]}/1/inf$ shows that K passengers are served simultaneously. The performance analysis includes; mean number of passengers in the system, mean waiting time of passengers in the system, mean length of queue and mean waiting time in the queue. The results obtained are compared to the ($M/M^{[K]}/1/inf$) models and ($D/D^{[K]}/1/inf$) queuing models. The rest of the paper is organized as follow. Section 2 shows the notations that are used throughout the paper. Section 3 describe a review to the PH distribution followed by abstraction of elevator service facility as queuing system in section 4. The performance parameters are described in section 5 and Section 6 illustrates the computational experiments and results. Finally, Section 7 closes the paper with the concluding remarks.

2. Notations

The notations used throughout the text is presented below:

μ	Elevator service rate in ped/h
1F	Ground floor
$(a_\circ, \alpha)$	Initial state probability vector
Q	Infinitesimal generator matrix
inf	Infinity
$\oplus$	Kronecker sum
$\otimes$	Kronecker product
-1F	Lower ground floor
E[N]	Mean number of passengers in the elevator service facility system
E[W]	Mean waiting time of passengers in the elevator service facility in minutes
$E[N_q]$	Mean number of passengers in the queue
$E[W_q]$	Mean waiting time of passengers in the queue in minutes
i	Number of passenger in the system
ε	Peak-hour factor
λ	Passenger arrival rate in ped/h
c^2	Squared coefficient of variation
π	Stationary probability vector
T	Transition Matrix
$h\ (1 \le h \le m)$	The phase of arrival process
$j\ (1 \le j \le n)$	The phase of service process

3. Concept of PH Distribution

Before, going into the details of $PH/PH^{[K]}/1/inf$ queuing system, the authors first give a brief introduction to PH distribution. PH distribution has been used in queuing modeling in the past because of its versatility and powerful applicability to the solution of various stochastic models that are based on Matrix analytic methods (Hu et al. 2013). Phase-type distribution was first introduced by the (Neuts 1981). The PH distribution is a probability distribution that is constructed by convolution of exponential distributions (Harchol-Balter 2013). It results from the system of one or more inter-related Poisson processes occurring in phases/sequence. The distribution is represented by a random variable that describe the time until absorption of Markov process with one absorbing state. Consider a continuous-time Markov Chain (CTMC) as shown in Fig.1 with state space $S = \{0,1,2,3,4\}$ where $1, 2, 3, 4$ states are the transient states represented by white circles and 0 is the absorbing state represented by a black circle. The λ represents rate of transition from one state to another state.

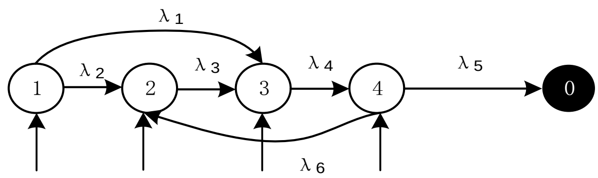

Fig. 1. CTMC representations of phase-type distributions

It has initial state probability vector $(a_{\circ}, \alpha) = (a_{\circ}, a_1, a_2, ...a_4)$ with $\sum_{i=1}^{4} a_i = 1$ and infinitesimal generator matrix Q given by Eq. (1);

$$Q = \begin{bmatrix} 0 & 0 \\ T^0 & T \end{bmatrix} \tag{1}$$

Where T is a $m \times m$ Matrix and rows in T Matrix must sum to 0 and also the authors have in Eq. (2);

$$T^0 = -T1 \tag{2}$$

Where 1 is a 4-dimensional column vector of 1's.

4. Abstraction of URTS elevator as a queuing system

In this paper, the authors are dealing with two-level (2 floors) URTS i.e., 1F and -1F. Passengers after passing through fare gates use elevator to move from IF to -1F where Platform is located. The focus in this research is one-way passenger traffic i.e., the queue at 1F in the down peak pattern. The elevator service begins when the elevator stops at 1F and ends at the time when elevator comes back to 1F as in the Fig. (2). There are two major phases in elevator operation i.e. Loading/unloading and movement. Elevator being a single server, its mean service time is the sum of all the times taken during loading, unloading, gate opening and closing, downward and upward movements.

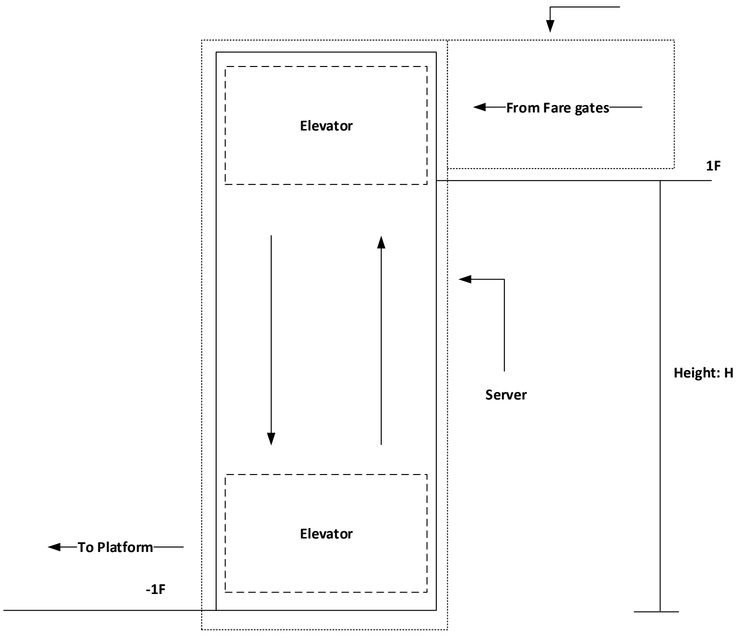

Fig. 2. Abstraction of URTS elevator as a queuing system

It is the round trip time that incorporates total time taken by a lift to complete one trip, from the main lobby to another floor of station back, during peak traffic in case of two-level URTS as illustrated by Fig. (3).

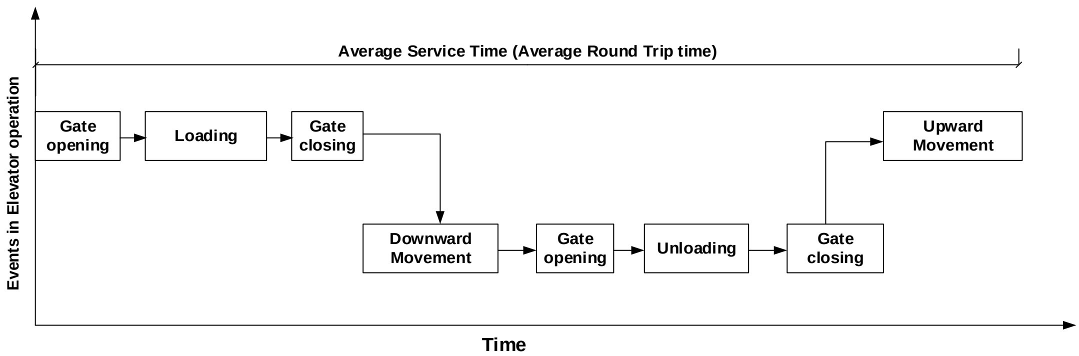

Fig. 3. Illustration of the mean service time of elevator

The round trip time between two floors can be computed by using Eq. (3) given by Liao et al. (2014);

$$T_{RT} = \frac{V_{max}}{a} + \frac{H}{V_{max}} \tag{3}$$

where:

T_{RT}	Round trip time
V_{Max}	Maximum uniform speed
a	Acceleration
H	Floor height

When the passengers arrive to the URTS elevator; they stay in the queue for sometimes before they are get served. In this paper, the authors fit PH distribution to the arrival interval of passengers and service time of elevator. The mean arrival rate (λ) and the squared coefficient of variation (c_a^2) during the peak hour can be obtained by using passenger flow 'q' and the peak hour factor ε from Eq. (4) and Eq. (5) given by (Jiang et al. (2015).

$$\lambda = \frac{q}{3600\varepsilon} \tag{4}$$

$$c_a^2 = \frac{e^{6.819\varepsilon}(\varepsilon - 1)^2}{4\varepsilon - 1} \tag{5}$$

5. Analysis of URTS elevator as PH/PH[K]/1/inf batch service queuing system

In order to fit PH distribution to the arrival rate of passengers and service time of elevator, it is considered that there are two parameters that describe arrival passenger traffic to the elevator i.e., λ_a and c_a^2 and two other parameters that describe elevator service process specified by μ_s and c_s^2. For the stability of this queuing system, the condition must be $\lambda < K\mu$. The PH distribution for arrival and services process is represented by initial probability vector $\boldsymbol{\alpha}$ and a Matrix $\mathbf{T}$. But there are four cases given by Sadre and Haverkort (2011);

1. If squared coefficient of variation c^2 for both arrival and service process is less than 1, a hypo-exponential distribution is used to fit arrival and service process with the number of phases given by $m = \frac{1}{c^2}$, initial probability vector $\boldsymbol{\alpha} = (1, 0, ..., 0)$ and the matrix T given by Eq. (6);

$$\mathbf{T} = \begin{pmatrix} -\lambda_0 & \lambda_0 & & & \\ & -\lambda_0 & \lambda_1 & & \\ & & .. & .. & \\ & & & -\lambda_{m-2} & \lambda_{m-2} \\ & & & & -\lambda_{m-1} \end{pmatrix} \tag{6}$$

Where;

$$\lambda_i = \frac{m}{E[X]}, \text{ for; } 0 \le i < m-2 \qquad \lambda_{m\text{-}1} = \frac{2m(1+\sqrt{\frac{1}{2}m(mc^2 - 1)}}{E[X](m+2-m^2c^2)}$$

$$\lambda_{m-2} = \frac{m\lambda_{m-1}}{2\lambda_{m-1}E[X] - m}$$

2. If c^2 is greater than 1 for both arrival and service process, a hyper-exponential distribution is used for fitting with the number of phases $m = 2$, where p is given by;

$$\mathrm{p} = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{\mathrm{c}^2\text{-}1}{\mathrm{c}^2+1}} \tag{7}$$

And the matrix T given by Eq. (8);

$$\mathbf{T} = \begin{pmatrix} \frac{-2p}{E[X]} & 0 \\ 0 & \frac{-2(1-p)}{E[X]} \end{pmatrix} \tag{8}$$

3. If c^2 is equal to 1, then the approximation corresponds to an Exponential distribution given by Hu et al. (2015)
4. If c^2 is very small, then the PH distributions with large number of states is obtained. For c^2 smaller than 10, the approximation corresponds to an Erlang-10 distribution given by Sadre and Haverkort (2011). Also c^2 smaller than 1/30 i.e. $c^2 \to 0$ produces generally results in approximation for the fixed-length distributions given by Hu et al. (2015).

If α and T are the parameters of PH arrival process that are obtained after fitting process and β and S are parameters for PH service process, then according to batch service discipline, single elevator can serve up to K passengers at a time. If the queue length of passenger is less than K, all passenger from the queue are served. With consideration of down-peak traffic only and First-come-first serve discipline (FCFS), each time during initiation of elevator service, first K passengers (in the form of batch) from the queue receive their service (Baba 1983). To analyze this system, the authors model the passenger-elevator system as $\mathrm{PH/PH^{[K]}/1/inf}$ batch service queuing system. $\mathrm{PH/PH^{[K]}/1/inf}$ is studied in term of continuous time Markov Chain (CTMC) with the state space $(0,i)$ where 0 is the state of service when it is idle $0 = \{(0,\mathrm{i}),\ 1 \le \mathrm{h} \le \mathrm{m}\}$. Since, i^{th} level represents i passenger in the system. Increase from level i to level $i+1$ represent an arrival process and decrease from level i to level $i-1$ $(i > 0)$ represent a departure process $\mathrm{i} = \{(\mathrm{i}, \mathrm{j}, \mathrm{h}),\ \mathrm{i} \ge 0,\ 1 \le \mathrm{j} \le \mathrm{n},\ 1 \le \mathrm{h} \le \mathrm{m}\}$. This process leads to an infinitesimal generator of Markov Chain given by a Matrix Q in Eq. (9);

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{B}_{00} & \boldsymbol{B}_{01} & & & & & & & & & \\ \boldsymbol{B}_{10} & \boldsymbol{A}_1 & \boldsymbol{A}_2 & & & & & & & & \\ & \boldsymbol{A}_0 & \boldsymbol{A}_1 & \boldsymbol{A}_2 & & & & & & & \\ & & \boldsymbol{A}_0 & \boldsymbol{A}_1 & \boldsymbol{A}_2 & & & & & & \\ & & & \ddots & \ddots & \ddots & & & & & \\ & & & & \ddots & \ddots & \ddots & & & & \\ & & & & & \boldsymbol{A}_0 & \boldsymbol{A}_1 & \boldsymbol{A}_2 & & & \\ & & & & & & \boldsymbol{A}_0 & \boldsymbol{A}_1 & \boldsymbol{A}_2 & & \\ & & & & & & & \boldsymbol{A}_0 & \boldsymbol{A}_1 & \boldsymbol{A}_2 & \\ & & & & & & & & \boldsymbol{A}_0 & \boldsymbol{A}_1 & \boldsymbol{A}_2 \\ & & & & & & & & & \ddots & \ddots \end{bmatrix} \quad (9)$$

Where;

$$B_{10} = I \otimes S^{\circ} \qquad B_{00} = T \qquad B_{01} = T^{\circ}\alpha \otimes \beta$$

$$A_2 = T^{o}\alpha \otimes I \qquad A_1 = T \oplus S \qquad A_0 = I \otimes S^{o}\beta$$

$$T^0 = \text{-}T1 \qquad S^0 = \text{-}S1 \qquad T \oplus S = T \otimes I + I \otimes S$$

Stability criteria of $PH/PH^{[K]}/1/inf$ is that $\rho < \frac{\lambda}{K\mu}$. The steady-state solution π of the Markov chain with generator matrix Q can be obtained by solution of global balance Eq. (10);

$$\pi Q = 0 \text{ and } \pi 1 = 1 \qquad (10)$$

Performance analysis of elevator service facility can be done by using infinite Quasi-birth-death process (QBD). The details of QBD are not given in details here but can be studied in Bolch et al. (2006) and (Stewart 2009). There are many numerical methods available including Matrix Geometric Method (MGM), Spectrum Expansion method, LU-Decomposition method, Gaussian Elimination method that are mentioned in Latouche and Ramaswami (1999) and (Stewart 2009). In this paper, the authors use MGM to solve the Eq. (10). The stationary probability vector π for Q is generally partitioned as $\pi = [\pi_0, \pi_1, \pi_2 ....]$. Solving $\pi Q = 0$ with the normalizing equation $\pi 1 = 1$ produces the following equations:

$$\pi_0 B_{00} + \pi_1 B_{10} = 0 \qquad (11)$$

$$\pi_0 B_{01} + \pi_1 A_1 + \sum_{i=1}^{K} \pi_i A_0 = 0 \qquad (12)$$

$$\pi_{j-1} A_2 + \pi_{j} A_1 + \pi_{j+K} A_0 = 0 \qquad (13)$$

$$\sum_{i=0}^{K} \pi_i 1 = 1 \tag{14}$$

The regular structure of Eq. (9) is a key to the efficient solution of QBD by MGM. The main idea behind this methods is that the stationary probability vector π has a matrix-geometric form, i.e., there exist a matrix R. For batch service process, the computation of R matrix is given by (Baba 1983). We can define the sequence of matrices $R(n)$ where $n \geq 0$. When $n = 0$ then $R(0) = A_2$ and when $n \geq 0$ then $R(n+1) = A_2 + R(n)A_1 + [R(n)]^{K+1} A_0$. If in case the stochastic matrix $A_{012} = A_0 + A_1 + A_2$ is irreducible, then $R(n)$ converges to the matrix $R \geq 0$ as in Eq.(15) that satisfies $sp(R) < 1$, where *sp* stands for spectral radius.

$$R = A_2 + RA_1 + R^{(K+1)} A_0 \tag{15}$$

And stationary probability vector satisfies as shown in Eq. (16),

$$\pi_{i+1} = \pi_i R, \quad i \geq 0 \tag{16}$$

In order to derive the vectors π_0 and π_1, it is required to solve the system of linear equations given by Eq. (17);

$$\pi_0 B_{00} + \pi_1 B_{10} = 0 \tag{17}$$

$$\pi_0 B_{01} + \pi_1 (A_1 + \sum_{i=1}^{K} R^i A_0) = 0 \tag{18}$$

By replacing π_2 with π_1 and writing equations in the form of matrix, the authors get Eq. (19):

$$(\pi_0, \pi_1) \begin{pmatrix} B_{00} & B_{01} \\ B_{10} & A_1 + \sum_{i=1}^{K} R^i A_0 \end{pmatrix} = (0, 0) \tag{19}$$

These equations are programmed in the MATLAB software to obtain rate matrix R and stationary probability vector π.

6. Performance analysis of elevator service facility at URTS

In order to compute stationary performance measures for the elevator service facility at URTS, the authors have the j^{th} - moment $E(N^j)$ of queue length distribution (including the passenger in service) given by Eq. (20);

$$E[N^j] = \sum_{i=1}^{\infty} i^j \pi_i 1 \tag{20}$$

Therefore; the mean number of passengers in elevator queuing system is given by:

$$E[N] = \sum_{i=1}^{\infty} i\pi_i 1 = \sum_{i=1}^{\infty} i\pi_1 R^{i-1} = \pi_1 \sum_{i=1}^{\infty} \frac{dR^i}{dR} = \pi_1 \frac{d\left(\sum_{i=1}^{\infty} R^i\right)}{dR}$$

$$= \pi_1 \frac{d\left(\left(I - R\right)^{-1} - I\right)}{dR} = \pi_1 \left(I\text{-}R\right)^{-2} \tag{21}$$

Using the stationary probability vector, the authors can compute the mean number of passenger in the elevator queuing system. Similarly; other performance parameters including mean waiting time in the system, mean number of passengers in the queue and mean waiting time the queue can be obtained by following equations respectively using little’s law;

$$E[N_q] = E\left[N\right] - \frac{\lambda}{K\mu} \tag{22}$$

$$E[W] = \frac{E\left[N\right]}{\lambda} \tag{23}$$

$$E[W_q] = \frac{E\left[N_q\right]}{\lambda} \tag{24}$$

7. Computational Experiments

This section illustrates the application of $PH/PH^{[K]}/1/inf$ batch service queue to the elevator service facility at URTS. Experiments are performed with various arrival rates of 100, 150, 200, 250, 300,350 400, 450 peds/h that corresponds to lower and higher arrival rates in peak hours at URTS. These arrival rates are less compared to other vertical transportation means theusage of elevator system is lower than the stairs and escalators at URTS. In this study, peak-hour factor is assumed to be 0.9. The calculated squared coefficient of variation from Eq. (2) is 1.78 and service rate squared coefficient of variation is taken as 1.2. Each time when the elevator is at 1F and gate opens, the service starts and first K passengers from the queue enter to elevator. Similarly; only one-way passenger down-peak traffic is considered in this research work so the passenger getting services only from 1F and -1F and not from -1F to 1F. After dismemberment from elevator, the empty elevator moves from -1F to 1F. Since K number of passenger are allowed to get elevator services simultaneously, so the authors use K = 6 and 8 depending upon the size of elevator and represent the accommodating capacity of elevator. K = 6 corresponds to accommodating 6 passenger with their respective luggage at a time. Similarly; K = 8 corresponds to elevators with large area that accommodate more passenger and their respective luggage usually at transfer stations where there is high passenger traffic intensity and elevator usage is high. Lee et al. (2009) have considered the inter-floor travel time as 2 seconds and uniform stop time of 6 seconds at each floor. But in case of URTS, we assume slightly higher values of loading and unloading time as passengers take more time due to carrying their luggage. The loading and unloading time of $2 \times 6 = 12$ seconds and $2 \times 10 = 20$ seconds are taken in this. Therefore; two different service times i.e. 32 seconds and 36 seconds are considered in the research due to variation in loading and unloading process of elevator.

The authors compare the results of performance analysis from $PH/PH^{[K]}/1/inf$ with markovian batch service queues and deterministic batch service queues on which existing design codes are based. The graphs are generated in MATLAB software by analysis of data. The Fig. (4)Illustratesperformance analysis of batch service queuing system when K = 6 and service time is 32 seconds. It is quiteobvious for all the performance parameters that there is increasing trend from lower to higher arrival rates. It is observed that for K = 6 and service time = 32configuration, the $D/D^{[K]}/1/inf$ line increase from 100 ped/h slightly and then become constant for higher arrival rates that corresponds to constant values of performance measure for all the value of arrival rates. With increase in coefficient of variation i.e., for $PH/PH^{[K]}/1/inf$ and $M/M^{[K]}/1/inf$ there is increase in the values of performance measure. The $PH/PH^{[K]}/1/inf$ gives higher value of all performance measures compared to $M/M^{[K]}/1/inf$ .Now if we see in Fig. (5), it is observed that configuration of K = 6 and Service time = 36, there is increase in all the values of performance measures. It is due to the fact that elevator with high service rate will process more and results in small queue length and waiting time.

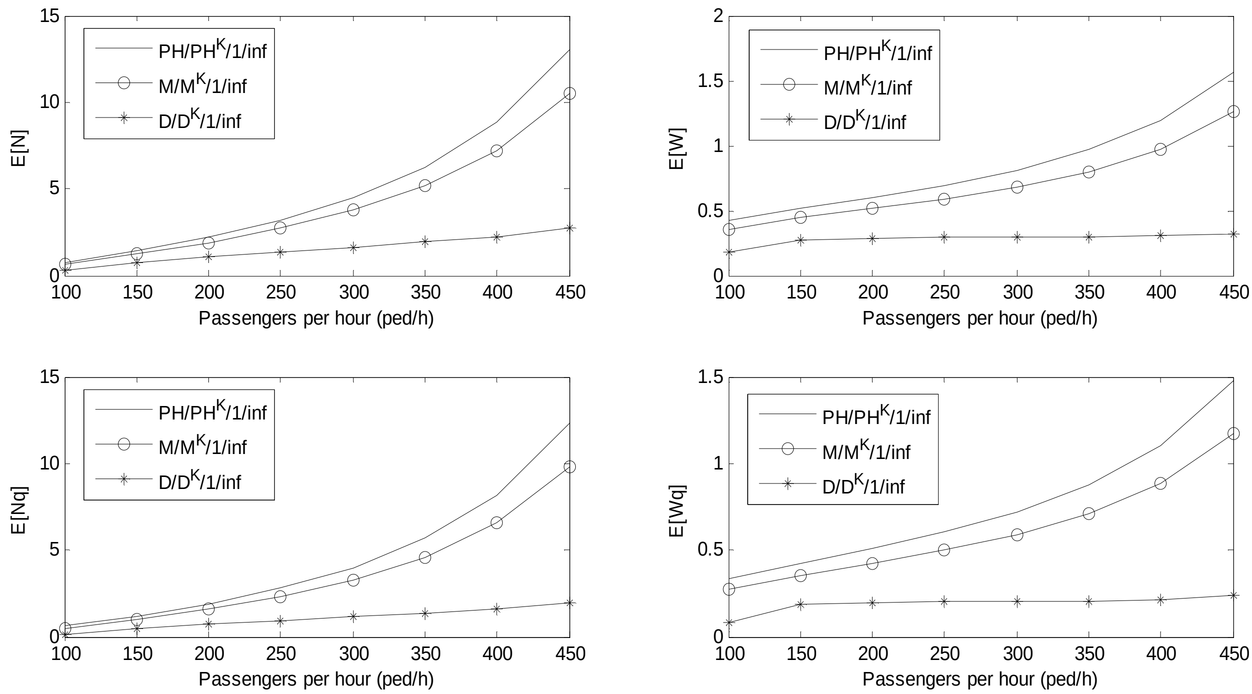

Fig.4. Performance parameters for K = 6 and Service Time = 32 seconds

Similarly; Fig. (6) and Fig.(7) with configuration K = 8 and service time 32 sec and 36 seconds respectively, the trend is same. It is observed that if the service time is kept constant i.e. 32 seconds then there is abrupt decrease in value of mean queue lengths and mean waiting time for K = 6 to K = 8. Also by keeping the K value constant, there is decrease in performance measures value from service time of 36 seconds to service time of 32 seconds. The results clearly indicates that existing design codes underestimate the results of performance measures. There is a need of improvement in design code by revising the analysis technique using PH distribution that accounts from variability in arrival and service processes.

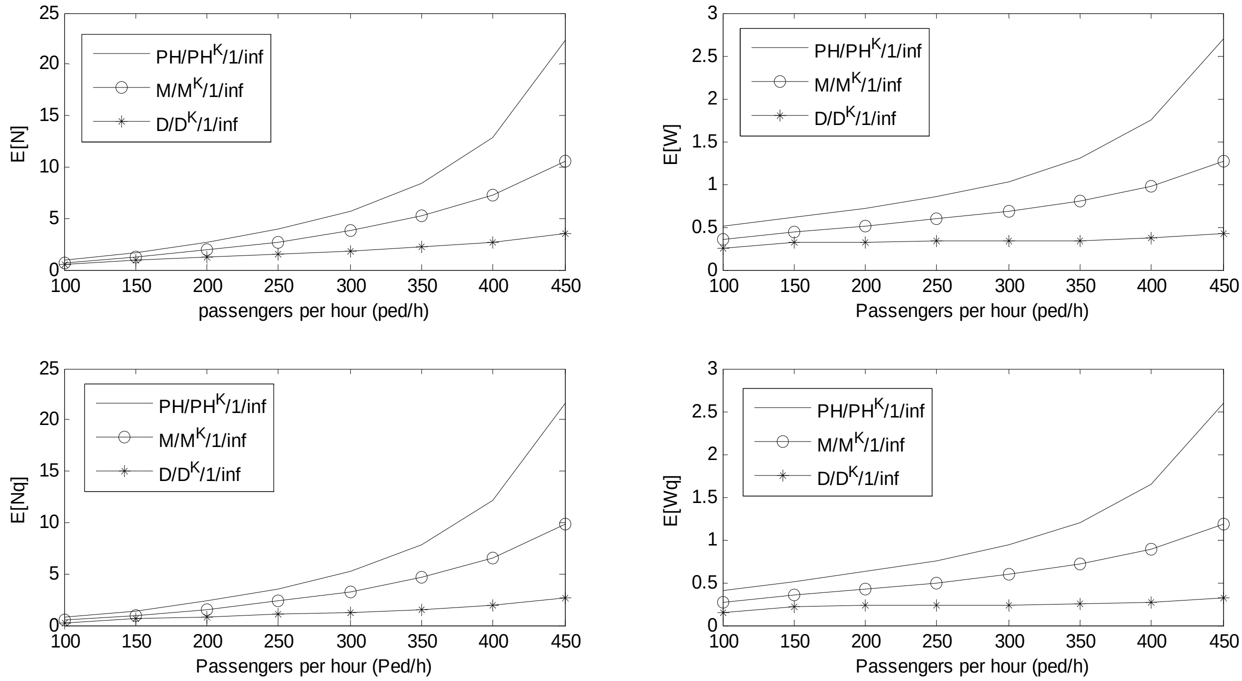

Fig.5. Performance parameters for K = 6 and Service Time = 36 seconds

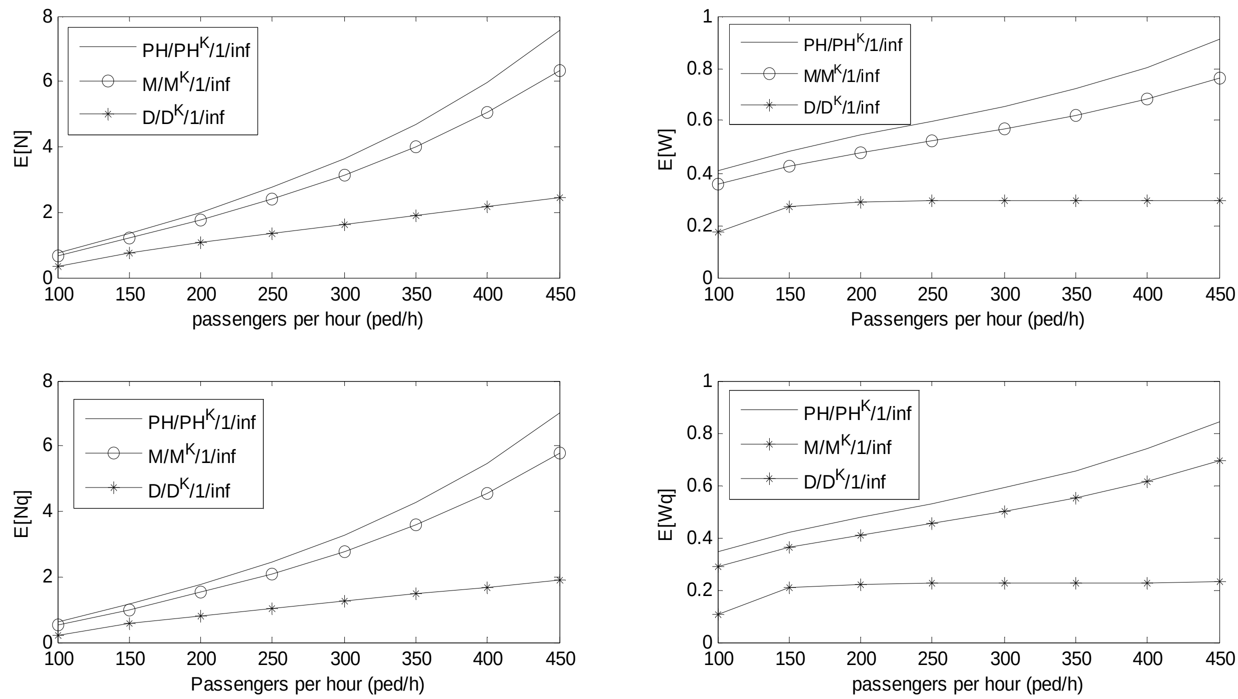

Fig.6. Performance parameters for K = 8 and Service Time = 32 seconds

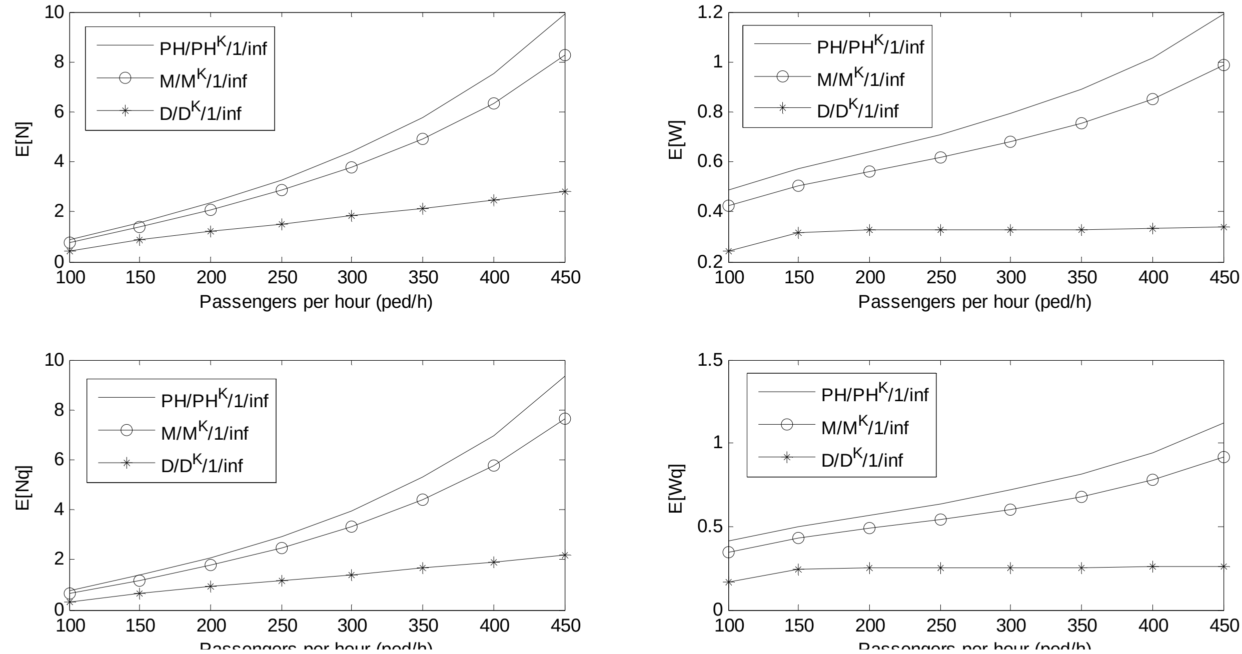

Fig.7. Performance parameters for K = 8 and Service Time = 36 seconds

8. Concluding remarks

In this study, performance measures for elevator service facility at URTS are obtained by $PH/PH^{[K]}/1/inf$ batch service queues, $M/M^{[K]}/1/inf$ and $D/D^{[K]}/1/inf$ queues.Further studies are recommended to obtain the performance measures of other service facilities at URTS by considering the finite capacity of service facilities. Simulation techniques can be used evaluate the service facilities and optimize the service facilities by using optimization techniques. This $PH/PH^{[K]}/1/inf$ batch service queuing model can be used in other transportation domains as well.

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